Detector-scrambler transition and information avalanche

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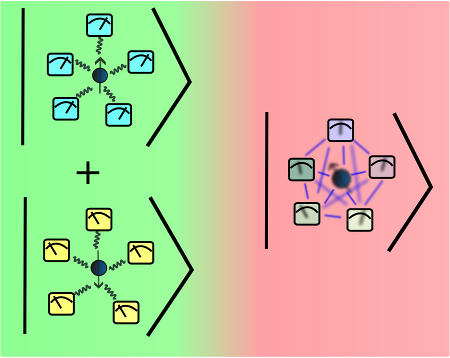
3èmes Journées du GDR IDE
see also: poster of Benoît Ferté

In a nutshell

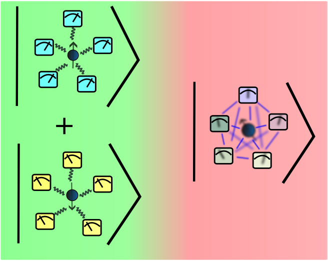
A thought-experiment where we turn a knob and make a measurement apparatus fail and become a "scrambler":



The failure takes place as phase transitions.

Motivation

- Statistical physics + quantum "foundation" questions = ?
- This work: emergence of classical objectivity as a "phase of information" where the latter propagates like a global avalanche.



The Heisenberg cut

An isolated system evolves under the Schrödinger equation

$$|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle \quad (1)$$

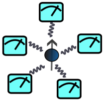
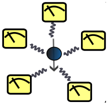
It is **deterministic** and **linear**.

When a macroscopic apparatus measures a quantum system, a **random** outcome is observed, accompanied by **non-linear** state update, e.g.:

$$\alpha|\uparrow\rangle + \beta|\downarrow\rangle \longrightarrow \begin{cases} |\uparrow\rangle & \text{w/ prob. } |\alpha|^2 \\ |\downarrow\rangle & \text{w/ prob. } |\beta|^2 \end{cases} \quad (2)$$

Wigner's friend scenario

Experimental evidence. Quantum theory remains successful in describing larger and larger systems.
The thought-experiment. Wigner has full quantum control over his friend's lab (a spin and a measurement apparatus).
For Wigner, the measurement process is unitary:

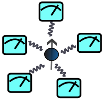
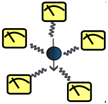
$(\alpha|\uparrow\rangle + \beta|\downarrow\rangle) \otimes |\text{app.}\rangle \to \alpha \Big|$  $\Big\rangle + \beta \Big|$  $\Big\rangle$ (3)

Quantum "ontology". The account of reality depends on the observer (or the H-cut).

Recent works: [Brukner], [Wiseman group], [Frauchiger-Renner], [Venkatesh], [Polychronakos], [Rovelli] . . .

Classical objectivity

For the friend, the measurement outcome is *objective*: It is retrievable from multiple records (computer, notebook, . . .) and can be agreed upon by many observers.
Wigner can attest the emergence of objectivity — know that his friend knows without knowing what she knows — by inspecting the coherent dynamics:

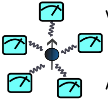
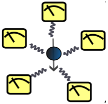
$\alpha|\uparrow\rangle + \beta|\downarrow\rangle \to \alpha \Big|$  $\Big\rangle + \beta \Big|$  $\Big\rangle$, (4)

which is an "information avalanche": the microscopic states $|\uparrow\rangle$ and $|\downarrow\rangle$ triggered distinct macroscopic responses of the lab. Many parts of the lab become correlated with the input.

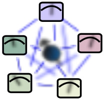
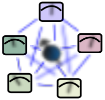
Quantum info. coaching of this statement: "Quantum Darwinism" by Zurek *et al.*

Classical objectivity *vs* Thermalisation

The "information avalanche" dynamics of the lab

$\alpha|\uparrow\rangle + \beta|\downarrow\rangle \to \alpha \, \big|$  $\big\rangle + \beta \, \big|$  $\big\rangle$ (5)

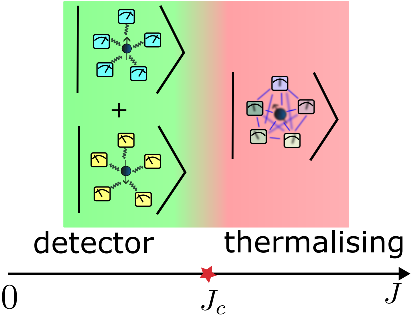
is very different from the thermalising one of a generic interacting quantum system:

$\alpha|\uparrow\rangle + \beta|\downarrow\rangle \to \alpha \, \big|$  $\big\rangle_1 + \beta \, \big|$  $\big\rangle_2$ (6)

The two states on the RHS are orthogonal but are locally indistinguishable! The initial information is "scrambled" (also called"encoded").

This talk

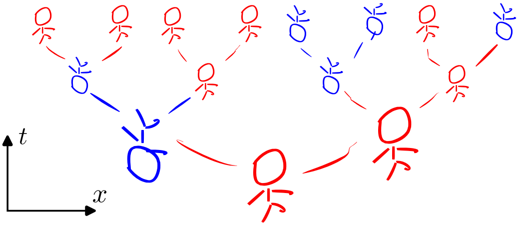
- Mean-field models of a sharp transition between the two behaviors.
- A “Harris criterion” to locate the transition in these models.



Much can be illustrated in a simple classical model.

Expansionist Chinese whispers (téléphone arabre)

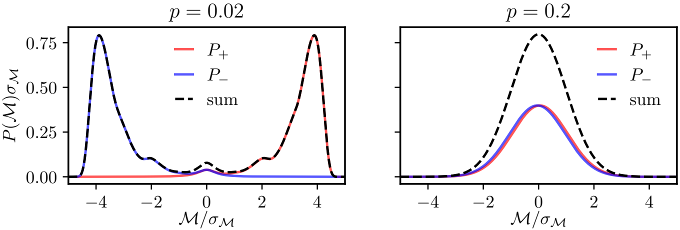
- Start with an initial player with a secret message $s = \pm 1$.
- Every player repeats the message to 2 new players . . .
- but has probability $p < \frac{1}{2}$ of hearing it wrong!



Can we infer the secret message ("input") by inquiring the last generation ("output") players?

Full counting statistics

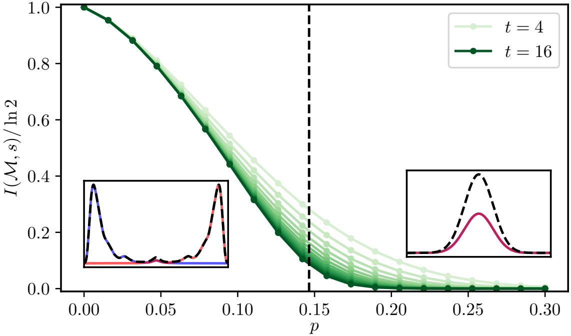
Consider the sum of the output messages/spins, $\mathcal{M}$. Below is its distribution conditioned on the secret message, $P_{s=\pm}$ $(t = 16)$:



- p small: non-Gaussian distribution, can infer the input.
- p large: Gaussian distribution, cannot infer the input (better than a random guess).

Phase diagram

We now compute the average amount of information that can be inferred (mutual information/conditional entropy):

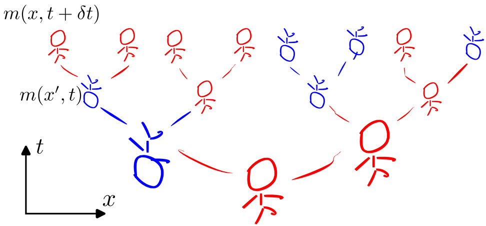


The data indicates a transition at some p_c where I vanishes.

Next

- We are going to predict exactly p_c, by deriving and applying a "Harris" criterion, valid for all models (quantum and classical) on an exponential expanding geometry.
- We will go from the specific model to a more general framework and make connection with real-space renormalization group (RG).
- This connection is well-known in the literature of tensor networks ("MERA", [Evenbly, G. Vidal, ...]), where the focus is numerical study of quantum critical states. We use the connection in a new and simpler way.

Consider the time evolution of the observable m measuring the local “spin”:



$$m(x, t+\delta t) = (1-p)m(x', t) + p(-m(x', t)) \tag{7}$$

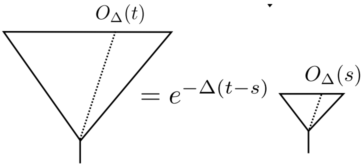
By convention, set $\delta t = \ln 2$ (so $N = e^t$). Then

$$\boxed{m(x, t+\delta t) = e^{-\Delta \delta t} m(x', t)\,,\ \Delta = -\ln(1-2p)/\ln(2)\,.} \tag{8}$$

We recognize a *scaling operator* (with dimension Δ), if we view the dynamics as implementing a real space RG.

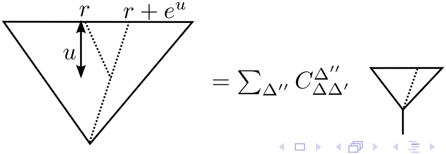
Reminders on RG

- A scaling operator O_Δ renormalizes simply when alone:



- **OPE:** If we evolve two (or more) of them, they will meet and merge:

$$O_\Delta(t,r)O_{\Delta'}(t,r+e^u) = \sum_{\Delta''} C_{\Delta\Delta'}^{\Delta''} e^{-u(\Delta+\Delta'-\Delta'')} O_{\Delta''}(t,r)$$



General argument (simplified)

Let the input be in state $|s\rangle$ (secret message). Can we infer it by measuring

$$\mathcal{O}(t) = \int O_\Delta(t, \mathbf{r}) d^d\mathbf{r}?$$

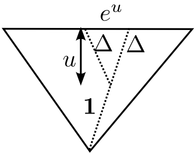
To find out, let us compare signal and noise:

- **Signal:** Mean value of $\mathcal{O}(t)$ conditioned on s

$$\langle \mathcal{O}(t) \rangle_s = e^{td} e^{-\Delta t} \langle s | O_\Delta(0) | s \rangle \tag{9}$$

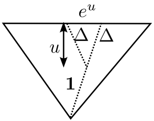
- **Noise:** Variance (assuming the OPE $\Delta\Delta \to \mathbf{1}$ dominates)

$$\left\langle \mathcal{O}(t)^2 \right\rangle \sim C^{\mathbf{1}}_{\Delta\Delta} e^{td} \int_{u=0}^{t} e^{-2\Delta u} e^{ud} du \tag{10}$$



$$\langle \mathcal{O}(t) \rangle_s^2 \sim e^{2td} e^{-2\Delta t}\,,$$

$$\left\langle \mathcal{O}(t)^2 \right\rangle \sim e^{td} \int_{u=0}^{t} e^{-2\Delta u} e^{ud} du$$



- $\Delta > d/2$: $u \sim 0$ dominates,

$$\langle \mathcal{O}(t) \rangle_s^2 \sim e^{2(d-\Delta)t} \ll \left\langle \mathcal{O}(t)^2 \right\rangle \sim e^{dt}$$

  Signal $\ll$ noise: no information can be inferred.
  Higher moments: show Gaussianity of $\mathcal{O}$.

- $\Delta < d/2$: $u \sim t$ dominates,

$$\langle \mathcal{O}(t) \rangle_s^2 \sim \left\langle \mathcal{O}(t)^2 \right\rangle \sim e^{2(d-\Delta)t}.$$

  Signal $\sim$ noise, inference is possible. (Non-Gaussianity is generically expected with interaction.)

"Harris criterion"

An expanding dynamics can propagate a nonzero amount of the input information if and only if there is a non-identity scaling operator with scaling dimension

$$\Delta < d/2 \tag{11}$$

where d is the space dimension.

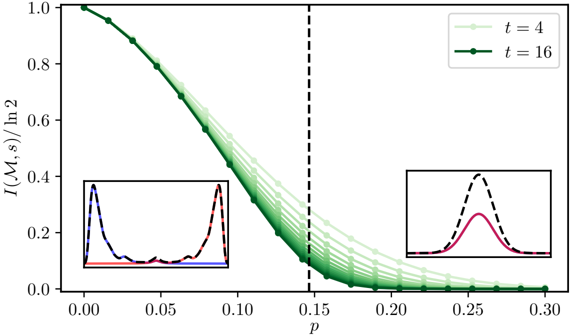
Remarks:

- Remains intact with spatial resolved probes.
- Resemblance to the Harris criterion (disordered perturbation), and that of divergence of order parameter fluctuation.
- When $\Delta < d/2$, same amount of information can be retrieved by inquiring any fixed small fraction f of the output, and thus accessible by a large number of observers $1/f \gg 1$ (objectivity).

Application to the téléphone arabre

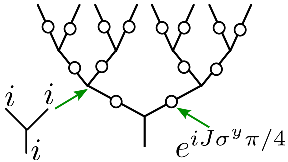
Recall $2^{-\Delta} = (1-2p)$. Since we embedded the tree in $d=1$,

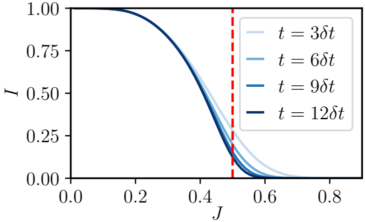
$$\Delta_c = \frac{1}{2} \implies 1 - 2p_c = \frac{1}{\sqrt{2}}\,,\ p_c = \frac{2-\sqrt{2}}{4} = 0.1464\ldots . \quad (12)$$



Application: "téléphone arabre quantique"

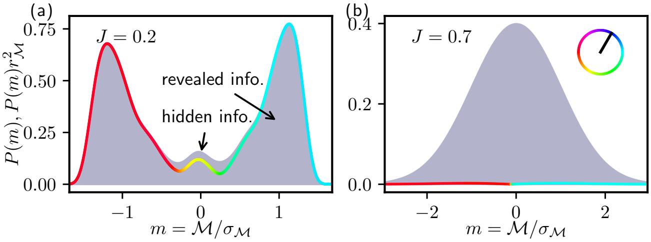
The players now copy qubits in the computational basis.
Stochastic errors are replaced by a unitary rotation sending $|0\rangle$ and $|1\rangle$ into coherent linear combinations thereof.
The criterion predicts exactly $J_c = 1/2$ (the dominating scaling operator is σ^z).





Aside: what's genuinely quantum?

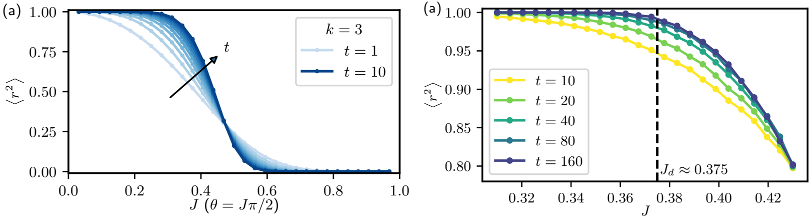
- To illustrate, consider again measuring the total spin $\mathcal{M}$:



- The best guess of the input qubit state, inferred from outcome, can be pointing anywhere in the Bloch sphere, not just the classical $|\uparrow\rangle, |\downarrow\rangle$.
- A proper discussion requires some quantum info setup. See [PRL 2024, PRA 2024].

A “purely quantum” transition

With fine-grained measure (but only accessing a small fraction of the output), there is another transition at $J_d < J_c$ where quantum mutual information becomes maximal.



More intricate (technically and physically), no known classical counterpart. Exact solvable example: [PRL 2024], “theory”: [PRA 2024].

Conclusion & Perspective

- Mean-field models & theory of "detector-scrambler" transition.
- Emergence of objectivity = "information avalanche" from microscopic to macroscopic.
- Information aspects of avalanches ("*Can we learn about its seed?*") will be relevant in more realistic models.
- Complex-system expertise of our GDR is needed for key quantum foundation issues, e.g., "*what is an agent/observer?*"