Yielding is an absorbing phase transition with vanishing critical fluctuations

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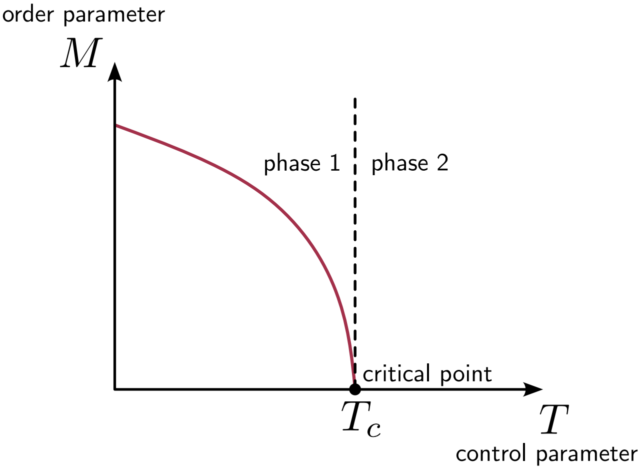
Continuous phase transitions



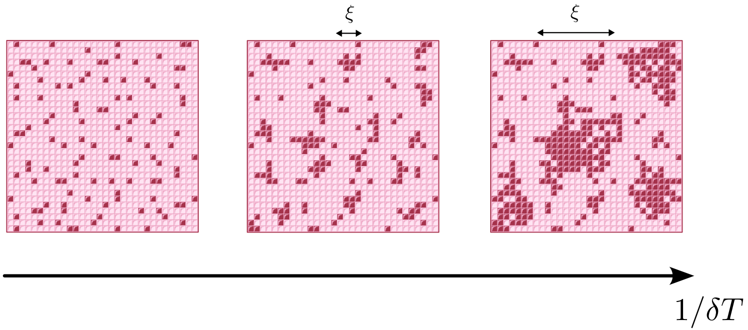
Definition

between two phases characterized by an **order parameter**, under the variation of a **control parameter**.

Examples

- Ferromagnetism
- Superconductivity
- Superfluidity

Critical behavior



The scaling hypothesis

ξ is the only relevant length scale.

- **scale-free** problem

Critical exponents

$$M \sim \delta T^{\beta}, \quad \xi \sim \delta T^{-\nu}$$
$$N \times (\Delta M)^2 \sim \delta T^{-\gamma'}$$

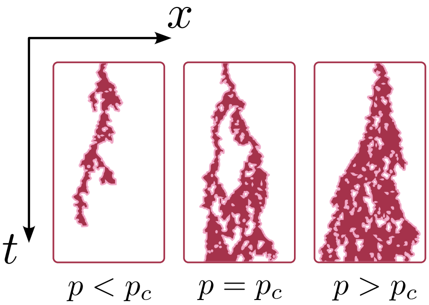
Absorbing phase transitions

- between an **active phase** and an **inactive phase** (absorbing state)

Absorbing state

can be reached by the dynamics but cannot be escaped

- e.g. epidemics models
- highly non-equilibrium



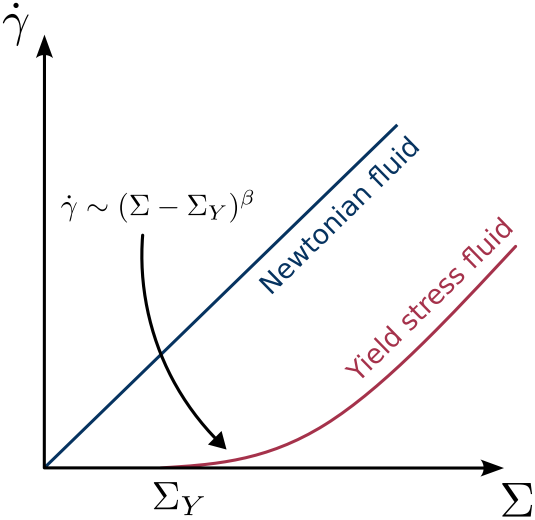
Directed percolation (DP)

- Unique absorbing state
- No special symmetry

Conserved directed percolation (CDP)

- Conserved field
- Infinity of absorbing states

A. Nicolas, E. E. Ferrero, K. Martens, and J.-L. Barrat, Reviews of Modern Physics 90, 045006 (2018)



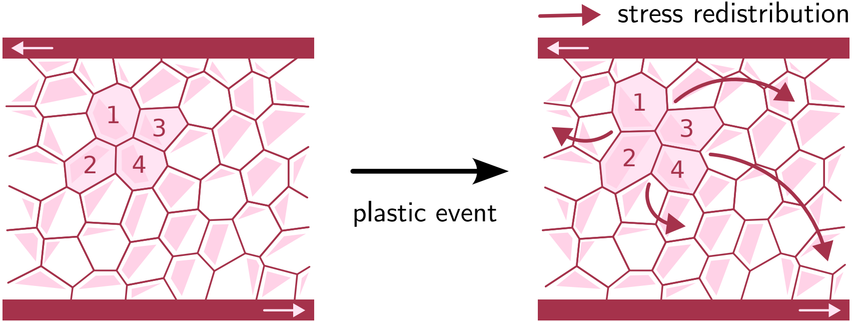
Examples

- clay
- sand
- toothpaste
- mayonnaise

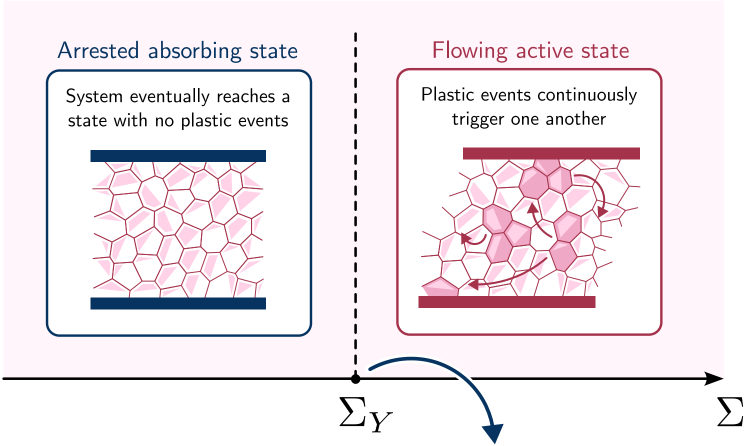
Yield stress

Below which material does not flow

- A plastic flow is a succession of local plastic events



1. Local **stress accumulation**
2. **Stress relaxation** and **local displacement** induced by plastic events
3. **Long range elastic redistribution** of the locally relaxed stress
4. **Triggering** of new plastic events



Order parameter : $\langle \dot{\gamma} \rangle$

Control parameter : Σ

Critical exponents

$\langle \dot{\gamma} \rangle \sim \delta\Sigma^{\beta}$ $\quad \xi \sim \delta\Sigma^{-\nu_{\perp}}$

$N(\Delta\dot{\gamma})^2 \sim \delta\Sigma^{-\gamma'}$

Discrepancy with CDP

Conserved directed percolation class

- Conserved field
- Infinity of absorbing states

2D-CDP exponents

$$\beta = 0.64 \quad \nu = 0.80 \quad \gamma' = 0.37$$

Discrepancy with CDP

Conserved directed percolation class

- Conserved field
  $\rightarrow$ local stress field $\{\sigma_i\}$
- Infinity of absorbing states
  $\rightarrow$ states that satisfy $\sigma_i < \sigma_Y, \quad \forall \sigma_i$

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2D-CDP exponents

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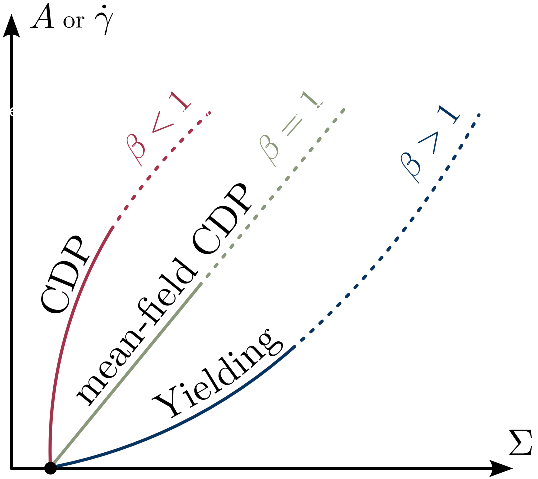
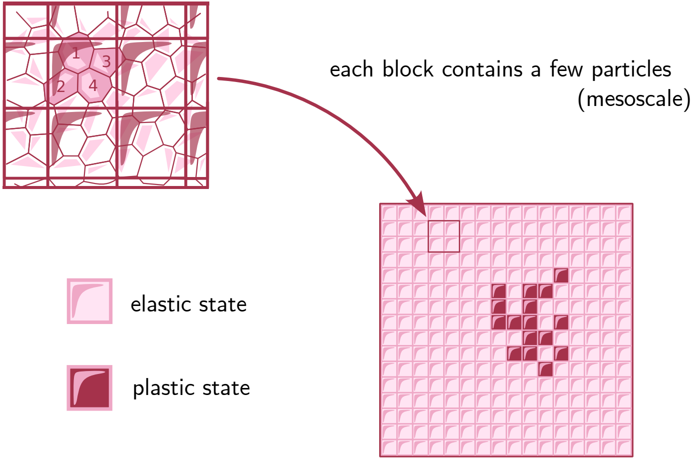
- elasticity induces **long range** interactions
- usually LRI tend to mean-field



Elastoplastic models

G. Picard, A. Ajdari, F. Lequeux, and L. Bocquet, Physical Review E 71, 010501 (2005)



- Three local variables $(\sigma_i, \epsilon_{\mathrm{pl},i}, n_i)$ with a model-dependent dynamics

Picard model

G. Picard, A. Ajdari, F. Lequeux, and L. Bocquet, Physical Review E 71, 010501 (2005)

Mechanical evolution

$$\partial_t \sigma_i = \sum_j G_{ij} \partial_t \epsilon_{\mathrm{pl},j}, \quad \partial_t \epsilon_{\mathrm{pl},i} = n_i \sigma_i$$

$$\text{avec} \quad G(|\mathbf{r} - \mathbf{r}'|) = \frac{\cos(4\theta)}{\pi |\mathbf{r} - \mathbf{r}'|^2}$$

State evolution

$$\begin{cases} n_i : \quad 0 \xrightarrow{\tau} 1 & |\sigma_i| > \sigma_Y \\ n_i : \quad 0 \xleftarrow{\tau} 1 & \forall \sigma_i \end{cases}$$

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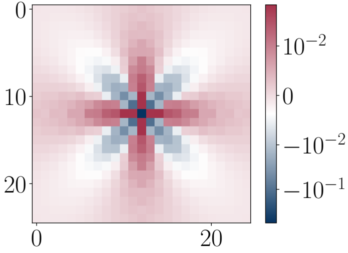

Picard model

B. Tyukodi, S. Patinet, S. Roux, and D. Vandembroucq, Physical Review E 93, 063005 (2016)
G. Picard, A. Ajdari, F. Lequeux, and L. Bocquet, Physical Review E 71, 010501 (2005)

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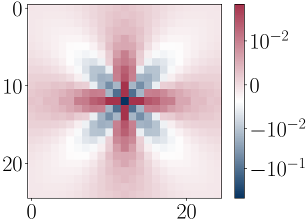

Presence of zero-mode

$$\int \mathrm{d}x \; G(\mathbf{r}) = \int \mathrm{d}y \; G(\mathbf{r}) = 0$$

- 0 cost for lines of activity

Picard model

Mechanical evolution

$$\partial_t \sigma_i = \sum_j G_{ij} \partial_t \epsilon_{\mathrm{pl},j}, \quad \partial_t \epsilon_{\mathrm{pl},i} = n_i \sigma_i$$

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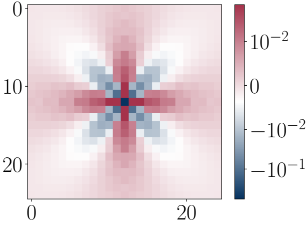

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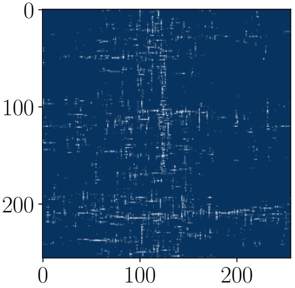


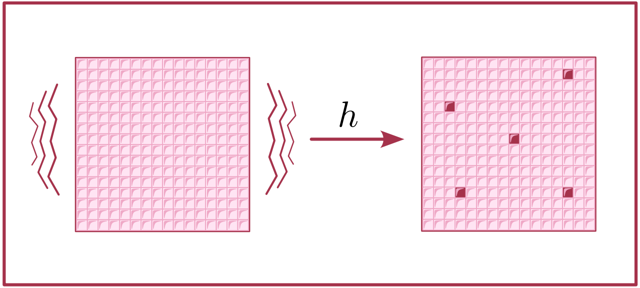
Insights from APT: getting close to the critical point

- Finite-size systems are absorbed even for $\Sigma > \Sigma_Y$.

Activation field

$$n_i : 0 \xrightarrow{h} 1 \quad \forall \sigma_i$$

S. Lübeck, International Journal of Modern Physics B 18, 3977 (2004)



New critical exponent

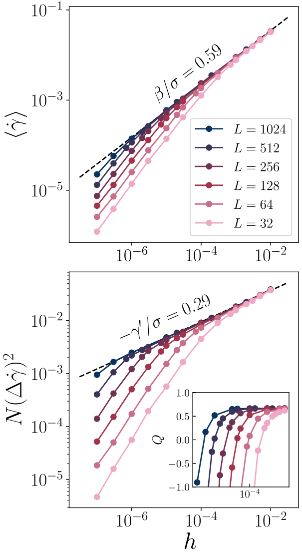
$$\dot{\gamma}(\Sigma = \Sigma_Y) \sim h^{\beta/\zeta}$$

$$N \times (\Delta\dot{\gamma})^2(\Sigma = \Sigma_Y) \sim h^{-\gamma'/\zeta}$$

- We can go as close as we want to the critical point
  - enables to probe the critical region

Finite-size scaling analysis

S. Lübeck, International Journal of Modern Physics B 18, 3977 (2004)



- FSE can be understood in the framework of the scaling hypothesis
  - **Idea**: introduce L as a parameter scaling as ξ

Universal functions

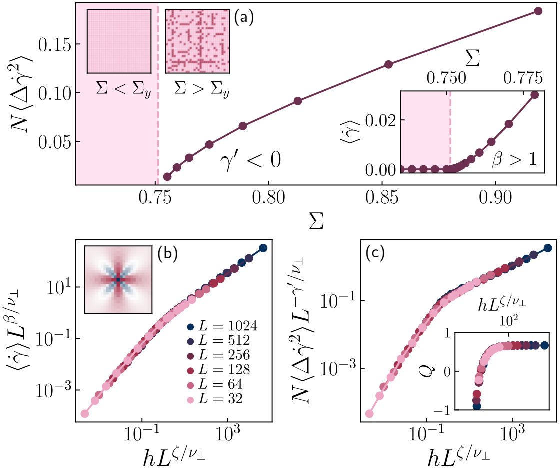
$$L^{\beta/\nu_\perp}\dot{\gamma} = F(0, L^{\zeta/\nu_\perp}h, 1)$$

$$L^{-\gamma'/\nu_\perp}N \times (\Delta\dot{\gamma})^2 = G(0, L^{\zeta/\nu_\perp}h, 1)$$

$$Q = 1 - \frac{\langle\dot{\gamma}^4\rangle}{3\langle\dot{\gamma}^2\rangle^2} = H(0, L^{\zeta/\nu_\perp}h, 1)$$

- Curves for all L should collapse for the right set $(\beta, \nu_\perp, \zeta, \gamma')$

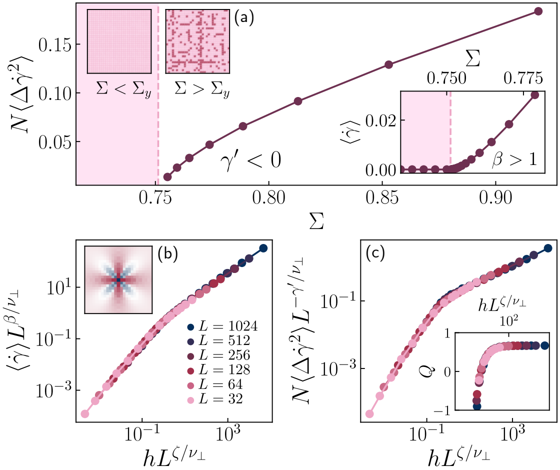
Critical behavior of plastic yielding



Estimated exponents

$$\beta = 1.5 \quad \nu = 1.1 \quad \gamma' = -0.70$$

Critical behavior of plastic yielding



Estimated exponents

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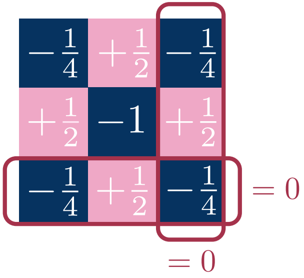
- **Fluctuations vanish** point
  - usually they diverge

Hyperscaling relation

$$2\beta + \gamma' = \nu_\perp d$$

- here $\frac{2\beta+\gamma'}{\nu_\perp d} \approx 1.02$



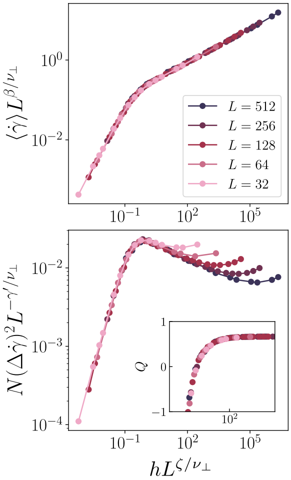
Estimated exponents

$\beta = 0.59$ $\nu = 0.70$ $\gamma' = 0.26$

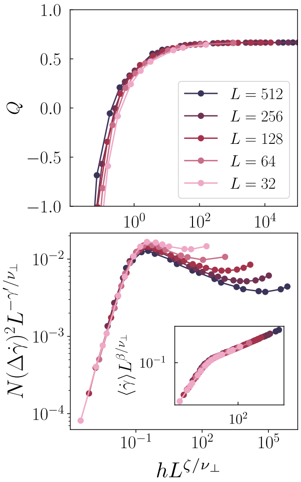
2D-CDP exponents

$\beta = 0.64$ $\nu = 0.80$ $\gamma' = 0.37$

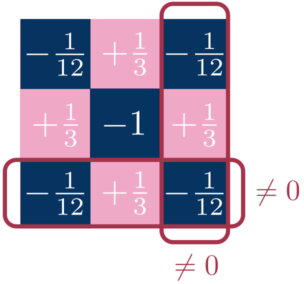
Best estimates



CDP exponents



Cutting off the long range interactions, importance of zero modes



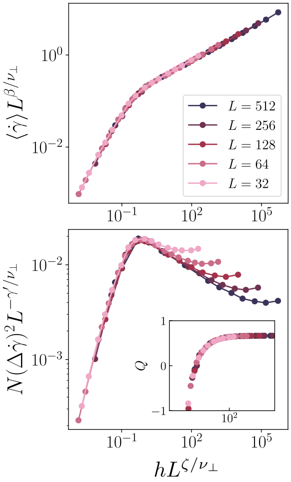
Estimated exponents

$\beta = 0.62 \quad \nu = 0.79 \quad \gamma' = 0.36$

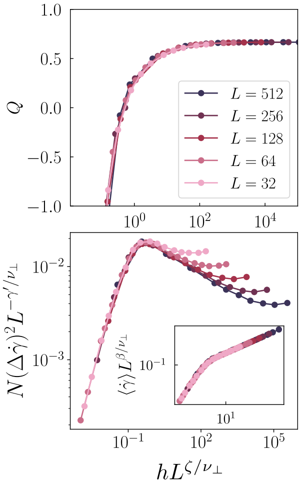
2D-CDP exponents

$\beta = 0.64 \quad \nu = 0.80 \quad \gamma' = 0.37$

Best estimates



CDP exponents



α-Picard model

- How do **long range interactions** affect the critical behavior ?

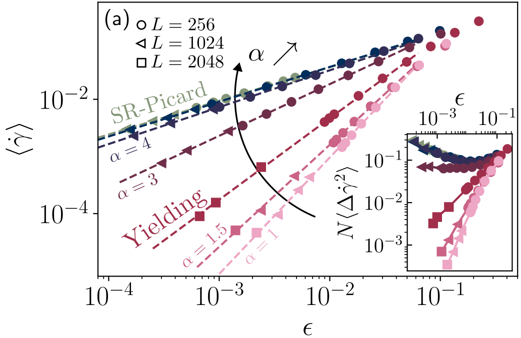
α-Picard propagator

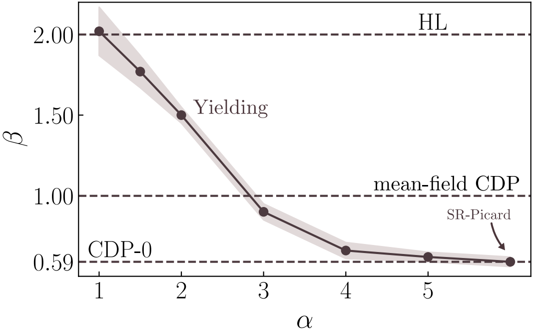
$$\mathcal{G}(r) \sim \frac{C_\alpha + \cos 4\theta}{r^\alpha}$$

α-Picard propagator

$$\mathcal{G}(r) \sim \frac{C_\alpha + \cos 4\theta}{r^\alpha}$$

- **Mean-field yielding** prediction (Hébraud-Lequeux) seems to be recovered for $\alpha = 1$
- **Yielding** is in a zone where critical behavior depends on α
- $\alpha = 4$ separates two regimes





Field equations for the CDP class

S. Lübeck, International Journal of Modern Physics B 18, 3977 (2004)

Equation on the activity field $A(\mathbf{r}, t)$

$$\partial_t A(\mathbf{r}, t) = (\kappa\rho(\mathbf{r}, t) - \xi)A(\mathbf{r}, t) - \lambda A^2(\mathbf{r}, t) + D_A \Delta A(\mathbf{r}, t) + \mu\sqrt{A(\mathbf{r}, t)}\eta$$

Equation on the conserved field $\rho(\mathbf{r}, t)$

$$\partial_t \rho(\mathbf{r}, t) = D_\rho \Delta A(\mathbf{r}, t)$$

Field equations for the CDP class

Equation on the plasticity field $A(\mathbf{r}, t) \sim \dot{\epsilon}_{pl}(\mathbf{r}, t)$

$$\partial_t A(\mathbf{r}, t) = (\kappa\sigma(\mathbf{r}, t) - \xi)A(\mathbf{r}, t) - \lambda A^2(\mathbf{r}, t) + D_A \Delta A(\mathbf{r}, t) + \mu\sqrt{A(\mathbf{r}, t)}\eta$$

Equation on the stress field $\sigma(\mathbf{r}, t)$

$$\partial_t \sigma(\mathbf{r}, t) = D_\rho \Delta A(\mathbf{r}, t)$$

Field equations for the CDP class

Equation on the stress field $\sigma(\mathbf{r}, t)$

$$\partial_t \sigma(\mathbf{r}, t) = D_\rho \Delta A(\mathbf{r}, t)$$

Field equations for the α-Picard model: importance of the 0-mode

mesoscopic scale

$$\partial_t \sigma_{i,j} = \sum_{k,l} G_{k,l} A_{i-k,j-l}(t)$$

macroscopic scale

Field equations for the α-Picard model: importance of the 0-mode

mesoscopic scale

$$\partial_t \sigma_{i,j} = \sum_{k,l} G_{k,l} A_{i-k,j-l}(t)$$

$\alpha < 6$

$\alpha > 6$

macroscopic scale

$$\partial_t \sigma = \int \mathrm{d}\mathbf{s}\, \mathcal{G}_\alpha(\mathbf{s}) \mathcal{F}[A]$$

$$\partial_t \sigma(\mathbf{r}, t) = K \partial_x^2 \partial_y^2 A(\mathbf{r}, t)$$

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$\alpha < 4$

$$\mathcal{F}[A] = \Delta A(\mathbf{r}, \mathbf{s}, t)$$

$\alpha > 4$

$$\mathcal{F}[A] = \Delta A(\mathbf{r}, \mathbf{s}, t) - \frac{s_\alpha s_\beta}{2} \partial_{\alpha\beta}^2 A(\mathbf{r}, t)$$

$\Delta A(\mathbf{r}, \mathbf{s}, t) = A(\mathbf{r} - \mathbf{s}, t) - A(\mathbf{r}, t)$

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$\alpha < 4$:

$$\mathcal{F}[A] = \Delta A(\mathbf{r}, \mathbf{s}, t)$$

LR

$\alpha > 4$:

$$\mathcal{F}[A] = \Delta A(\mathbf{r}, \mathbf{s}, t) - \frac{s_\alpha s_\beta}{2} \partial^2_{\alpha\beta} A(\mathbf{r}, t)$$

LR-0

$\alpha > 6$:

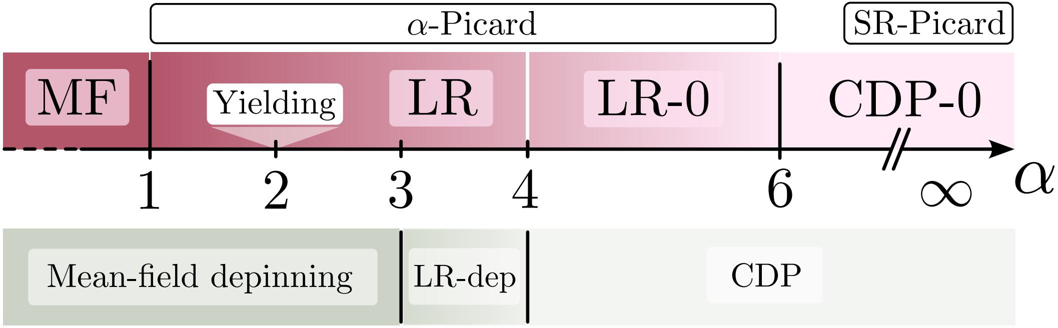
$$\partial_t \sigma(\mathbf{r}, t) = K \partial_x^2 \partial_y^2 A(\mathbf{r}, t)$$

CDP-0

$\Delta A(\mathbf{r}, \mathbf{s}, t) = A(\mathbf{r} - \mathbf{s}, t) - A(\mathbf{r}, t)$

Overview

M. E. Fisher, Collective Properties of Physical Systems (Academic Press, 1973), pp. 16–37



Conclusion

Results

- Plastic yielding is an absorbing phase transition from an **arrested absorbing phase** to a **flowing active phase**.
- Characterization of yielding leads to **unusual exponents** and vanishing critical fluctuations
- **Yielding differs from CDP** because of:
  - the presence of a zero-mode
  - **non-local** elastic interactions

T. Jocteur et al. Physical Review Letters: Yielding is an absorbing phase transition with vanishing critical fluctuations (forthcoming)

Perspectives

- How to consider **negative interactions** in the activity equation?
- Study of other convex phase transitions