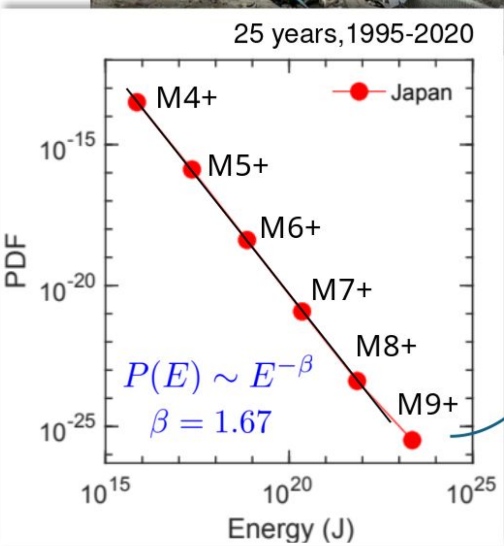
Criticality and predictability of scale invariant avalanches

K. Duplat, F. Detcheverry and O. Ramos

Scale invariant dynamic

Earthquakes



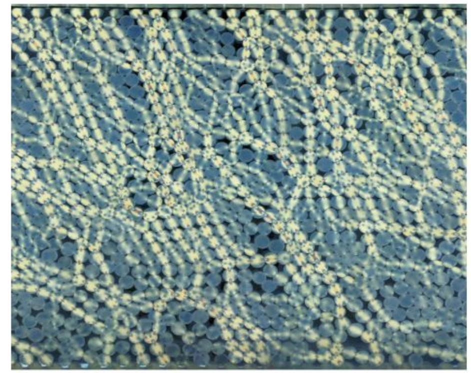
Tōhoku 2011

Labquakes in granular shear $\beta = 1.71$

Lherminier , PRL(2019)

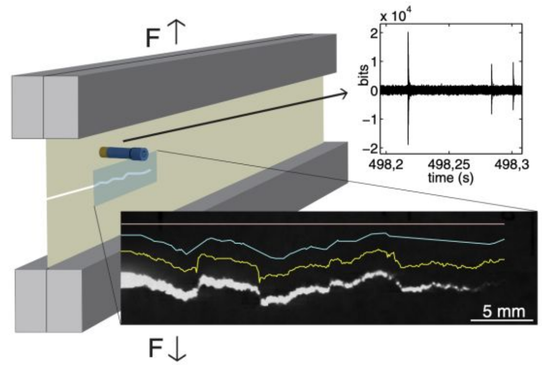
$\beta = 1.89$

Douin (2024)



Subcritical fracture $\beta = 1.51$

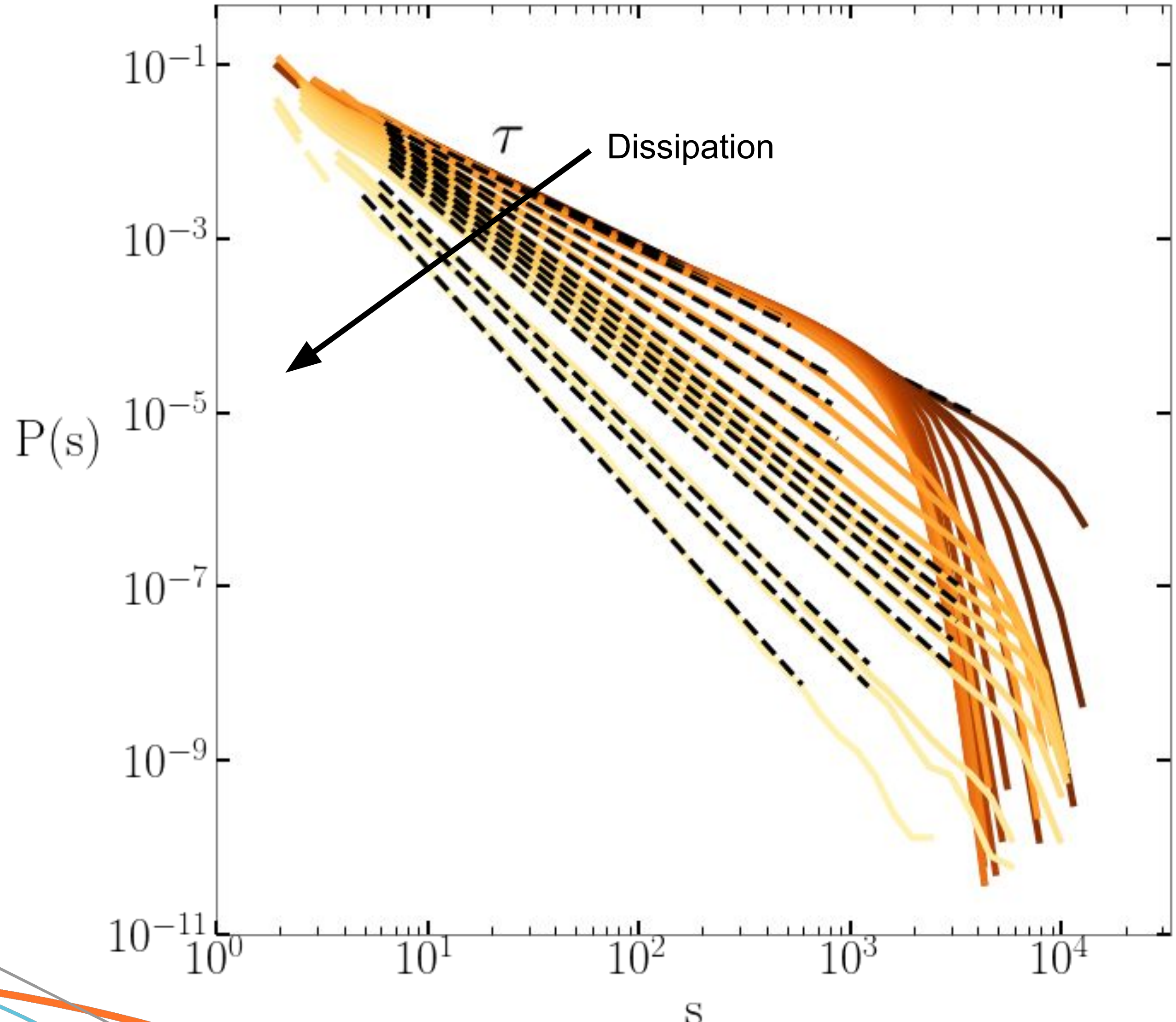
Stojanova PRL(2014)



Scale invariant phenomenon: Phenomenon with no characteristic size for an event.

OFC Model

Exponent variation with the dissipation

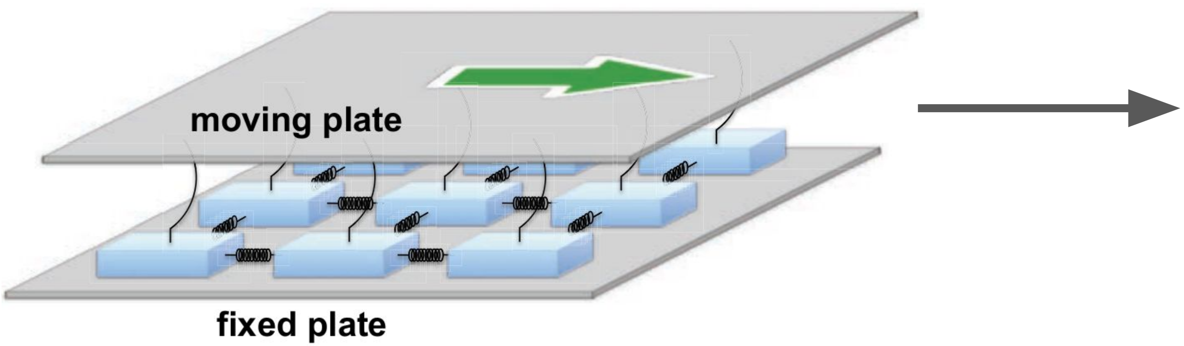


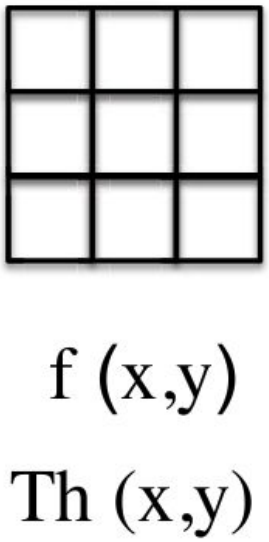
OFC model

Olami - Feder - Christensen model[1992]

Burridge et Knopoff model [1967]

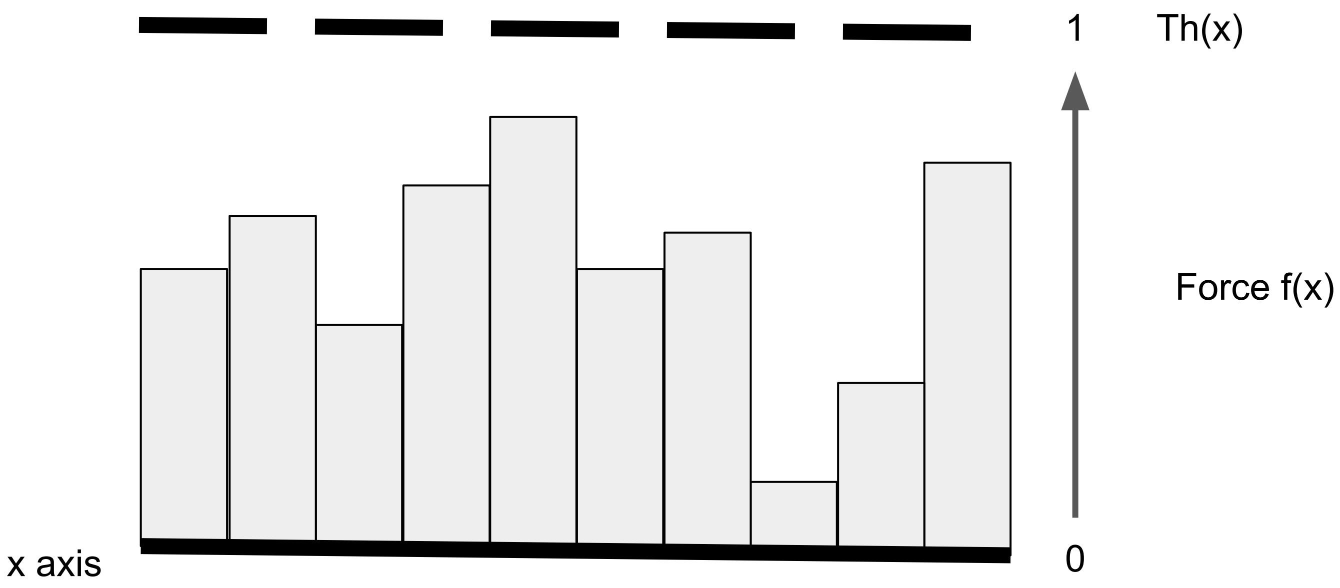
Cellular automaton (OFC)





OFC model

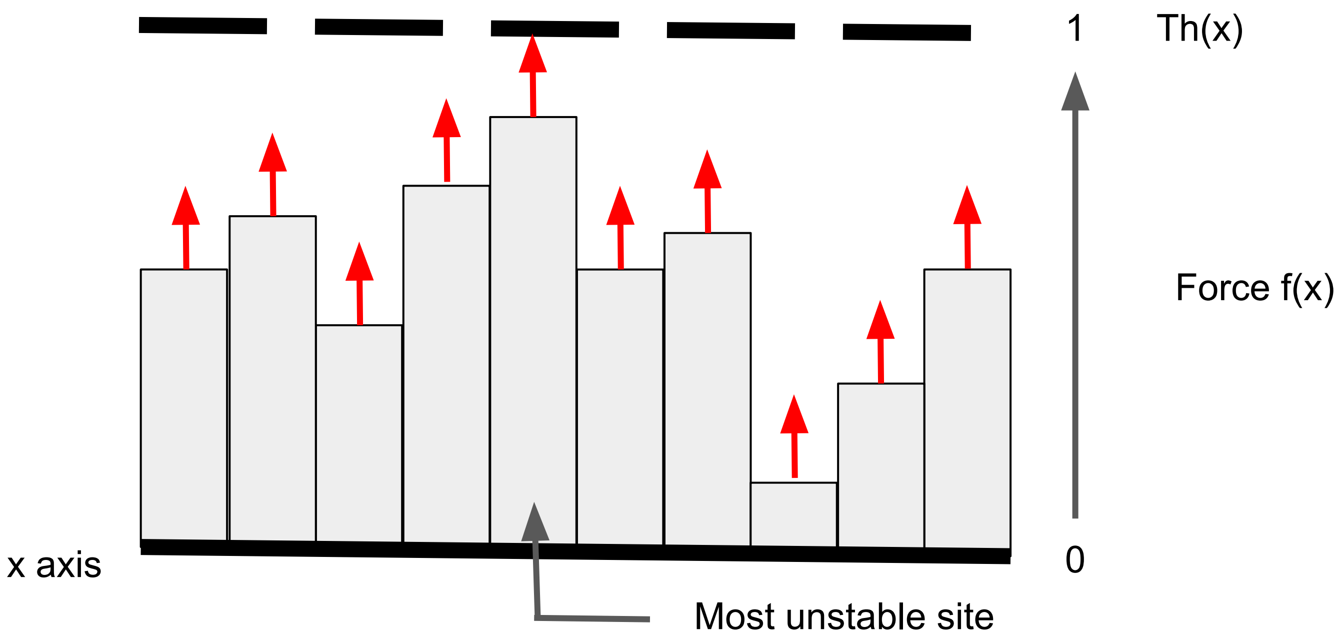
1D example



Initial state : sites are initialized at random values between 0 and 1

OFC model

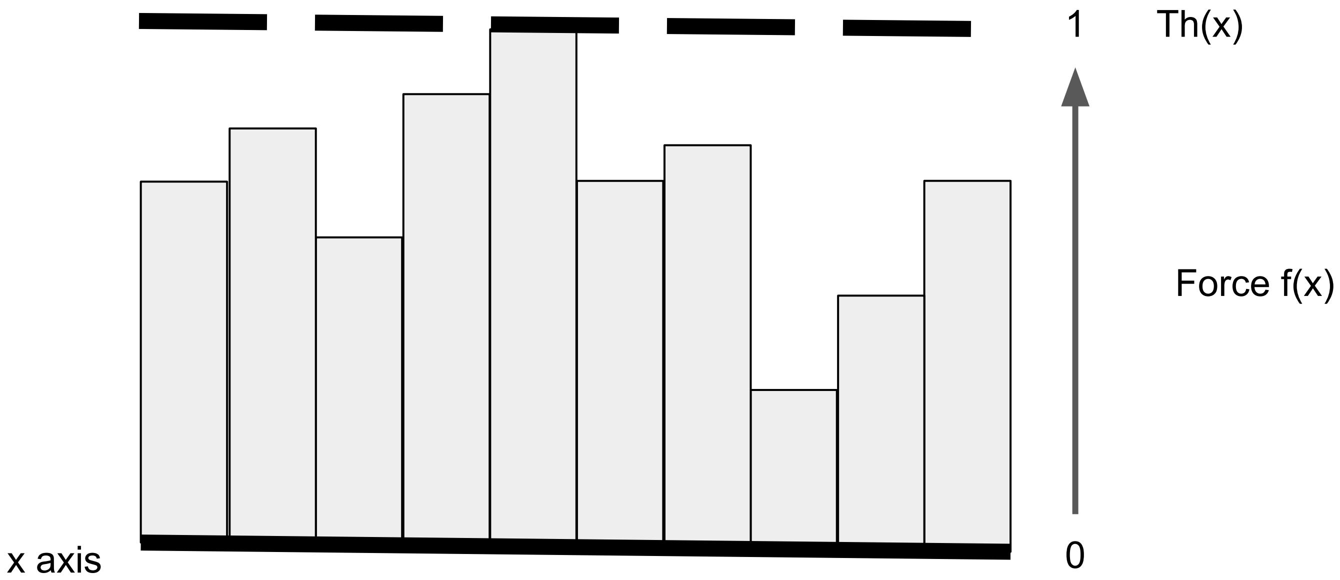
1D example



Energy incrementation in the entire system by the smallest deviation to the threshold

OFC model

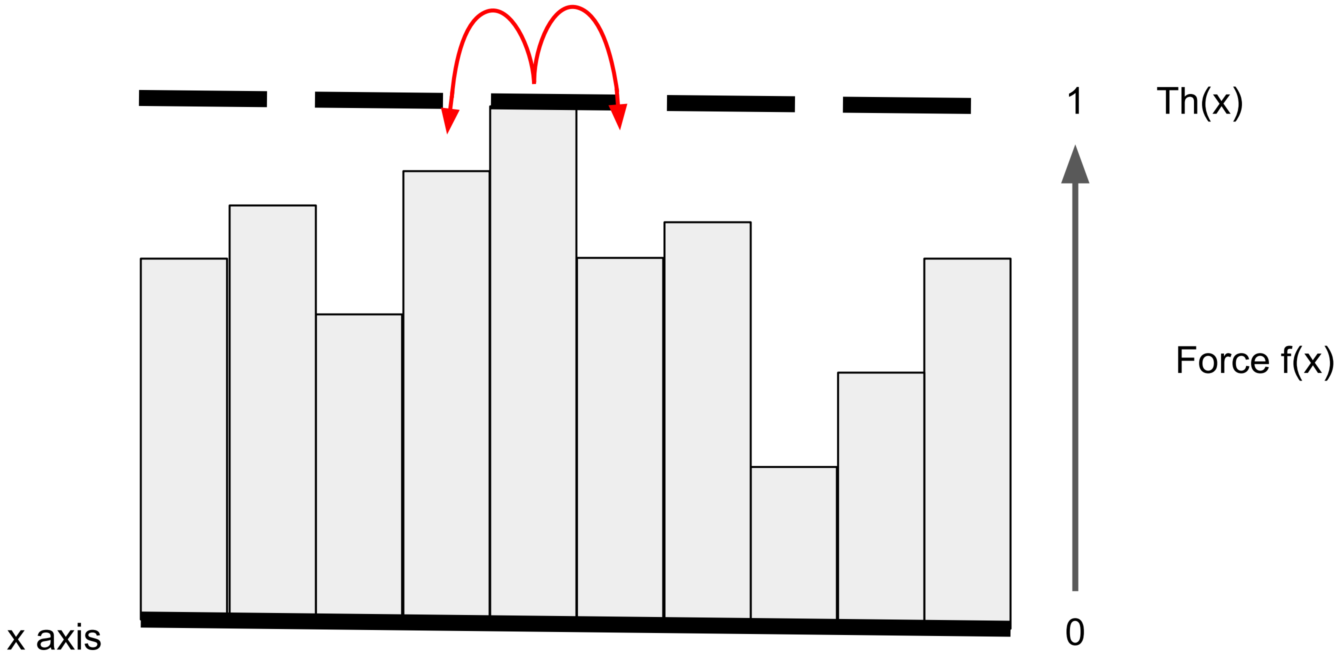
1D example



The most unstable site reaches its threshold and topple

OFC model

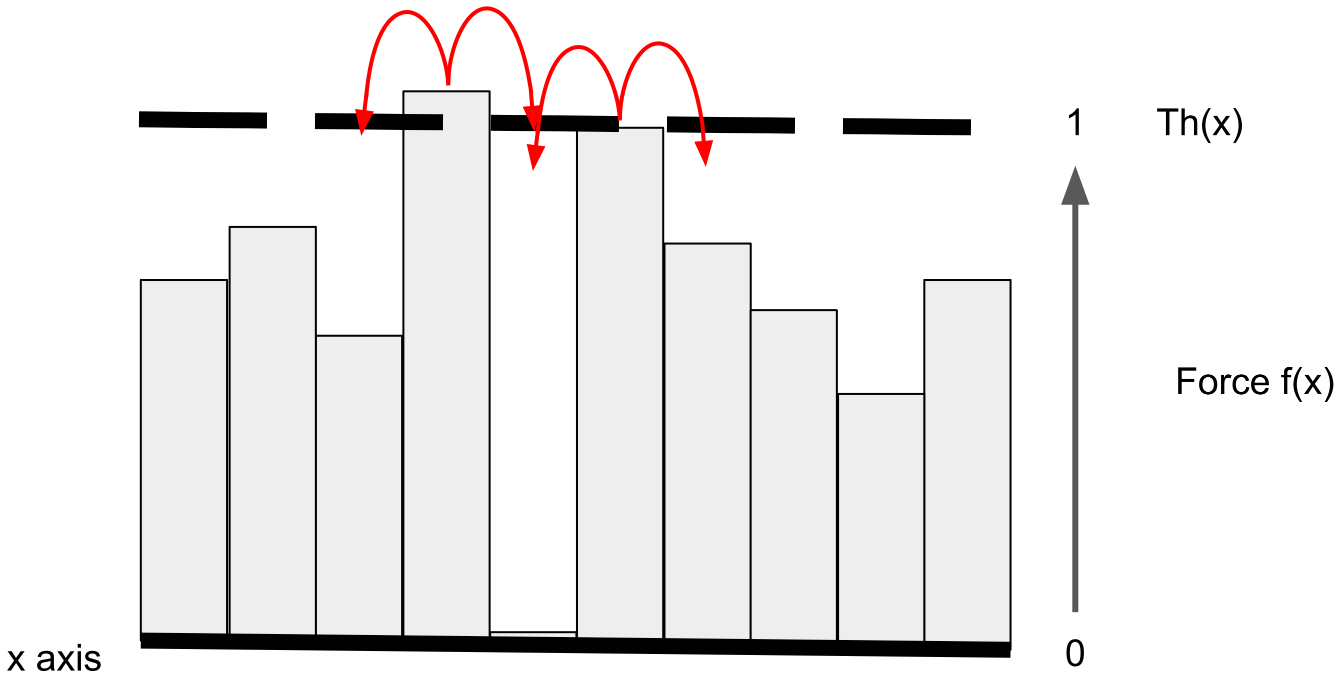
1D example



Redistribution of its energy to its neighbors

OFC model

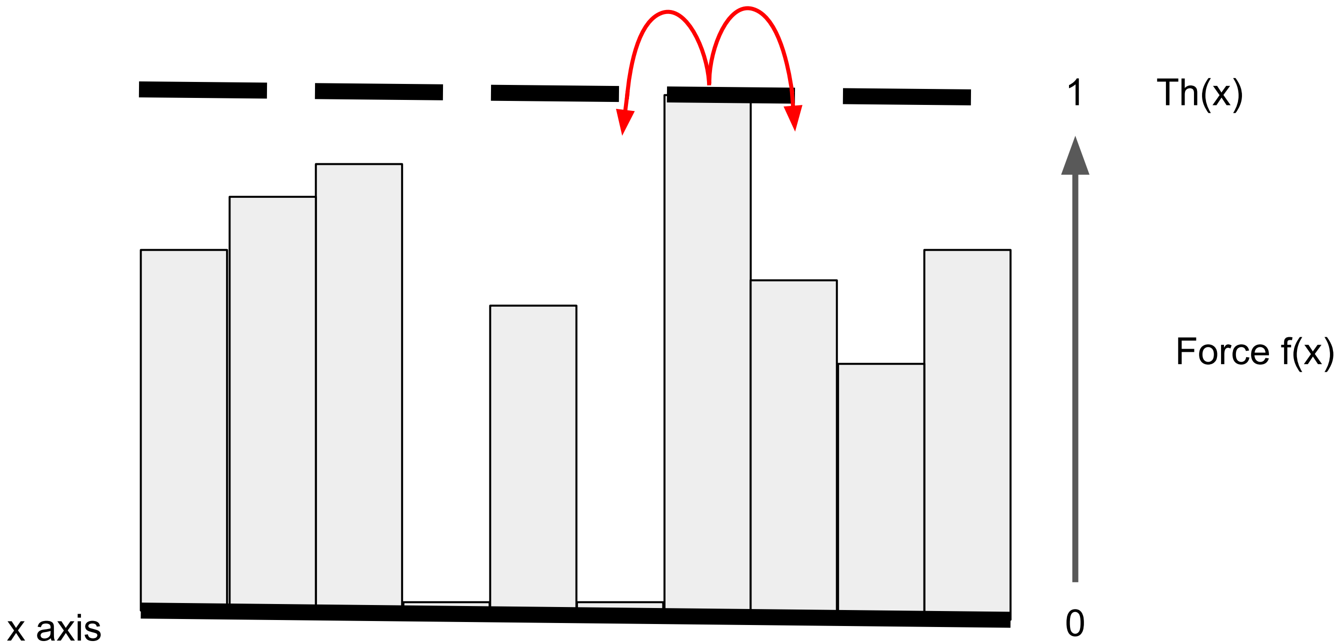
1D example



The process continues…

OFC model

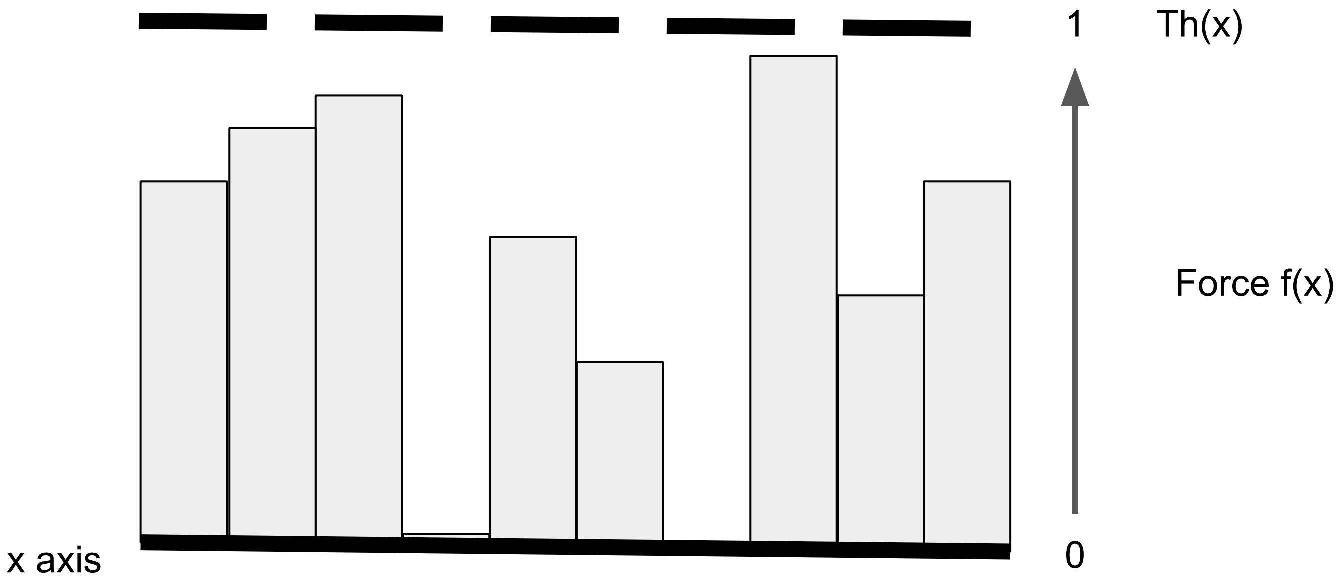
1D example



The process continues…

OFC model

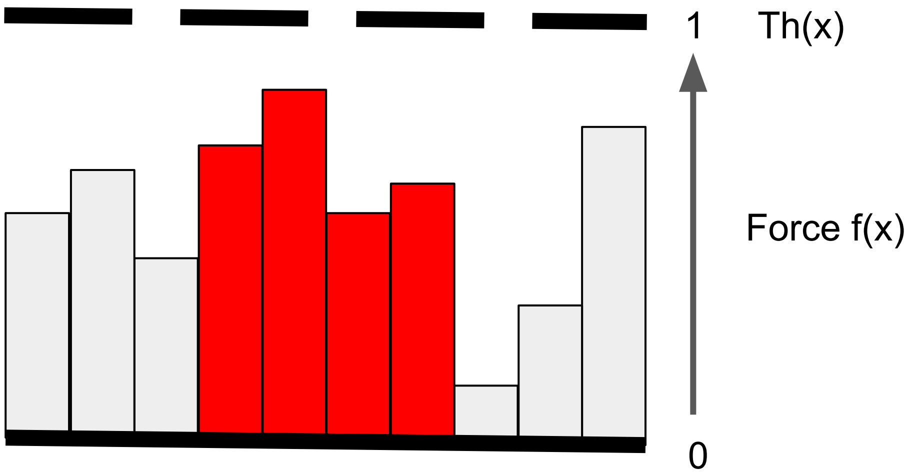
1D example

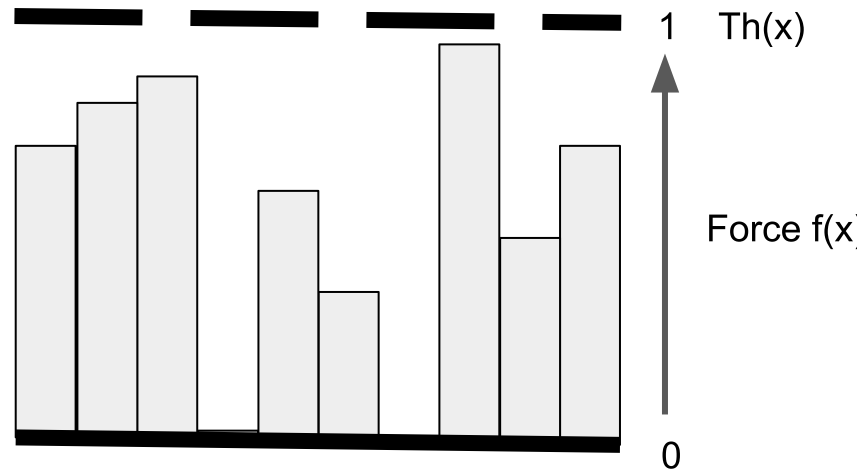


… until the system reaches a new stable state

OFC model

1D example





Avalanche : process starting from the toppling of the first site and ending when a new stable state is reached

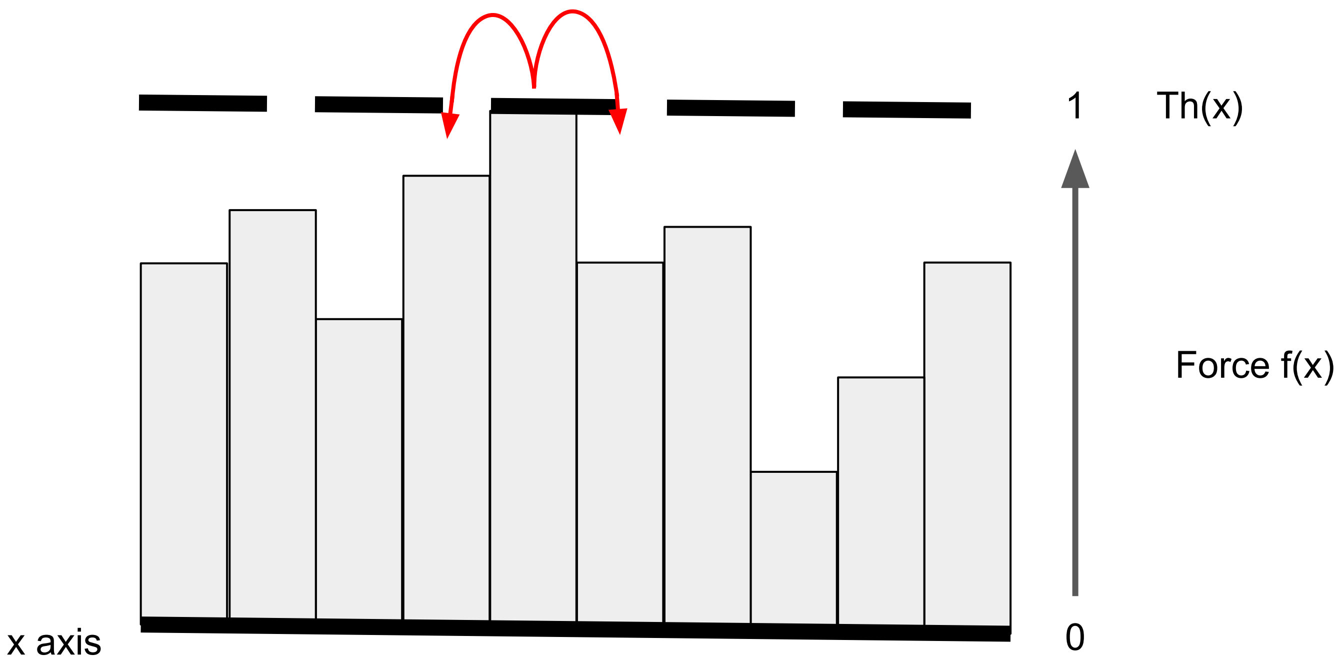
Avalanche size (s) : Number of sites that toppled

OFC model

Introduction to the dissipation

ν

$\nu = 0.5$



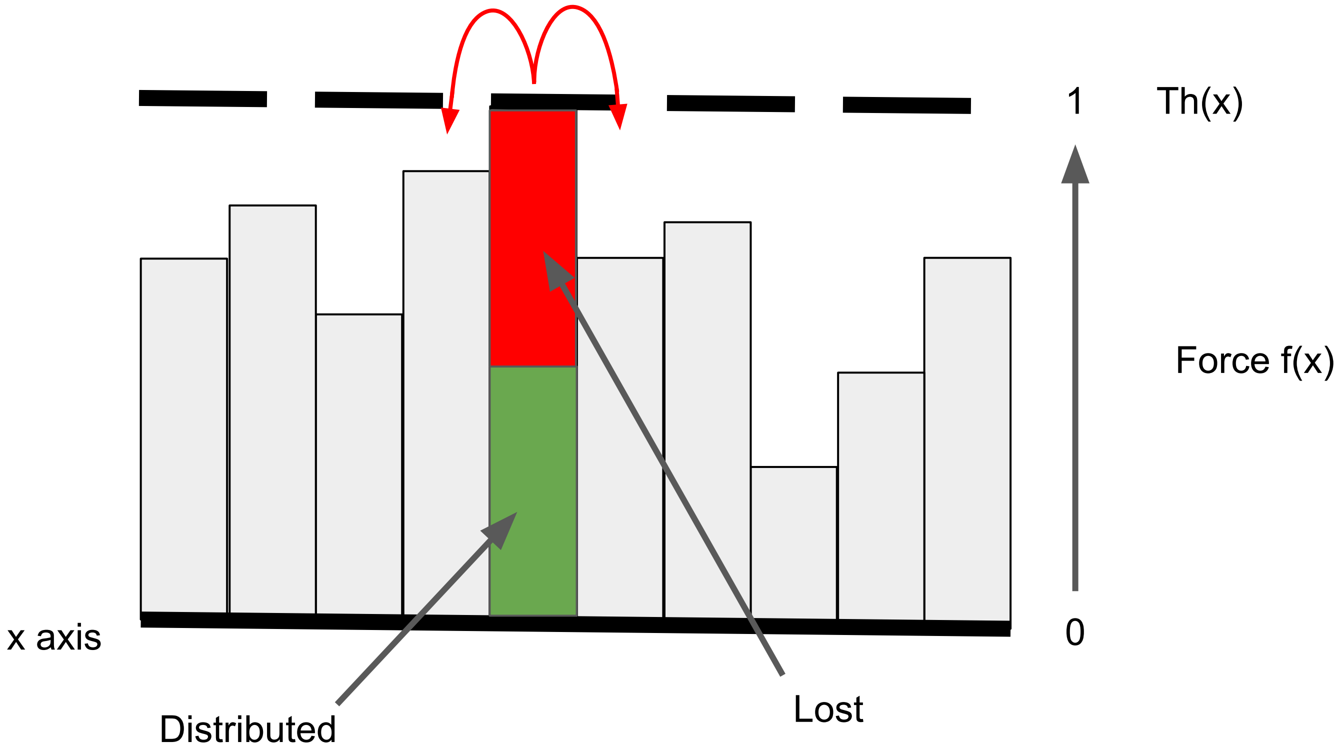
Dissipation : Energy lost during the toppling of a site

OFC model

Introduction to the dissipation

ν

$\nu = 0.5$



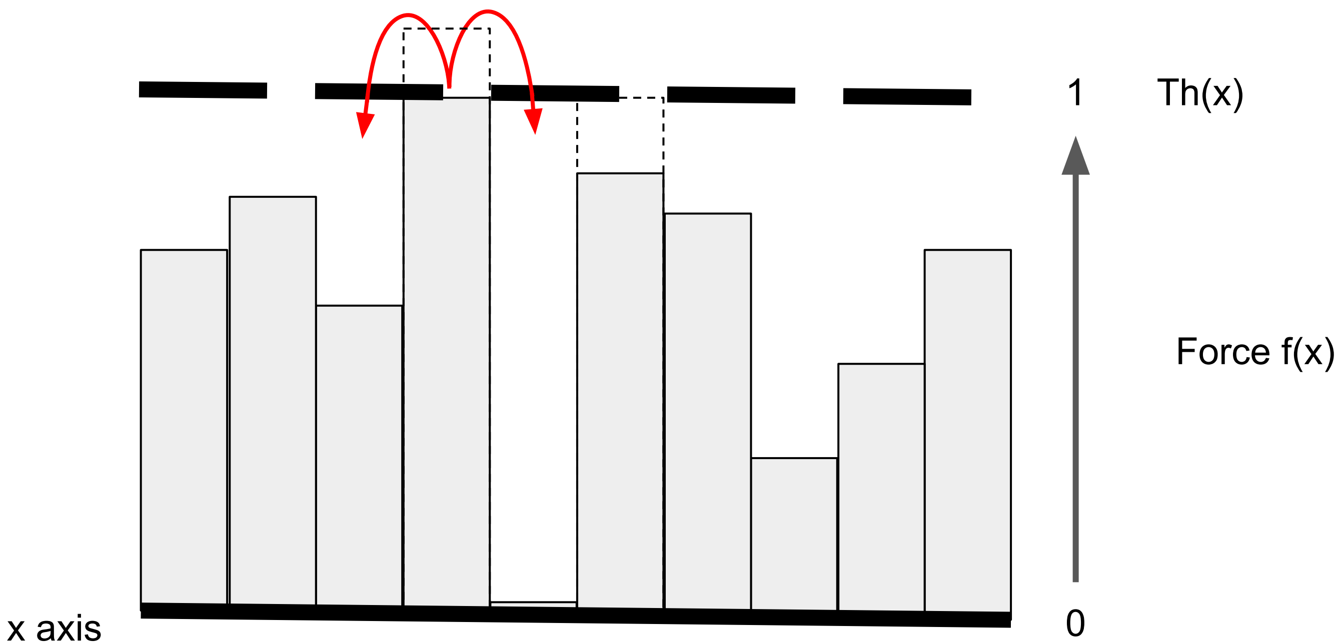
Only 50% of the force will be distributed

OFC model

Introduction to the dissipation

ν

$\nu = 0.5$



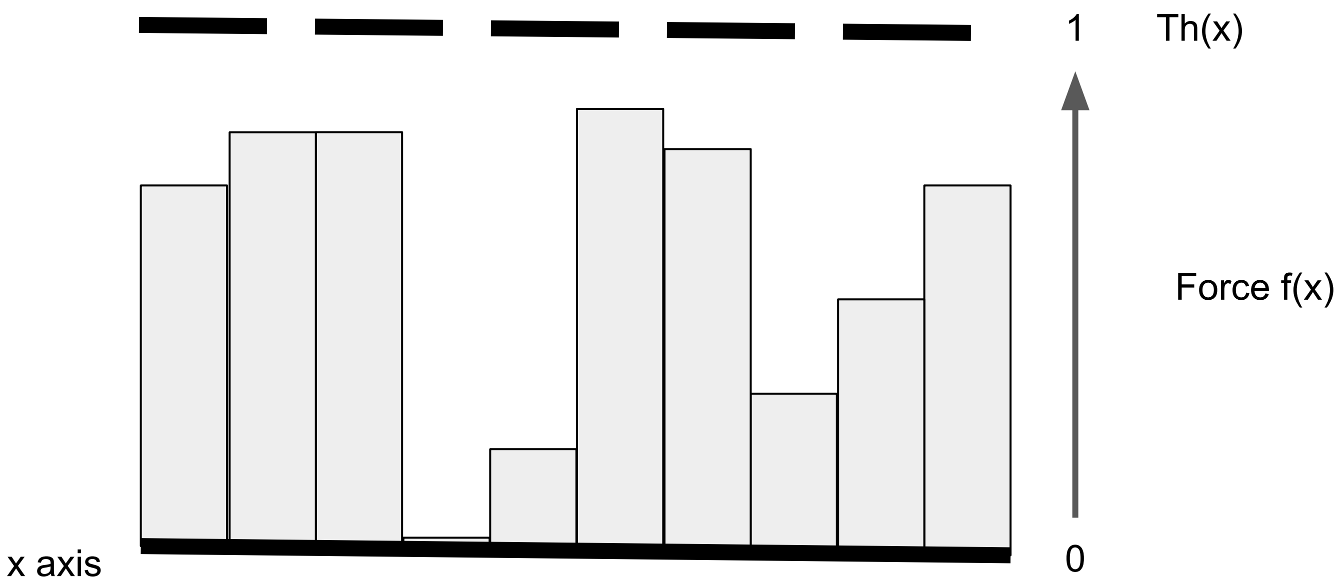
The process continues…

OFC model

Introduction to the dissipation

ν

ν = 0.5

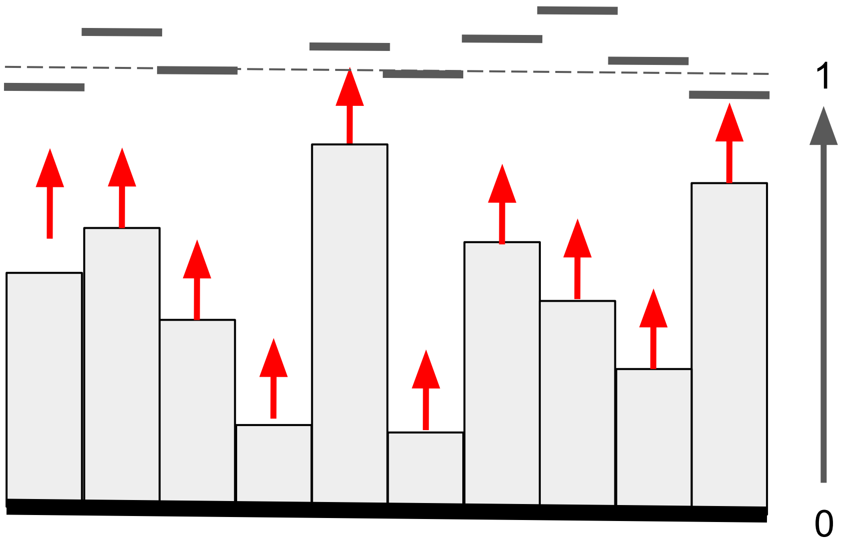


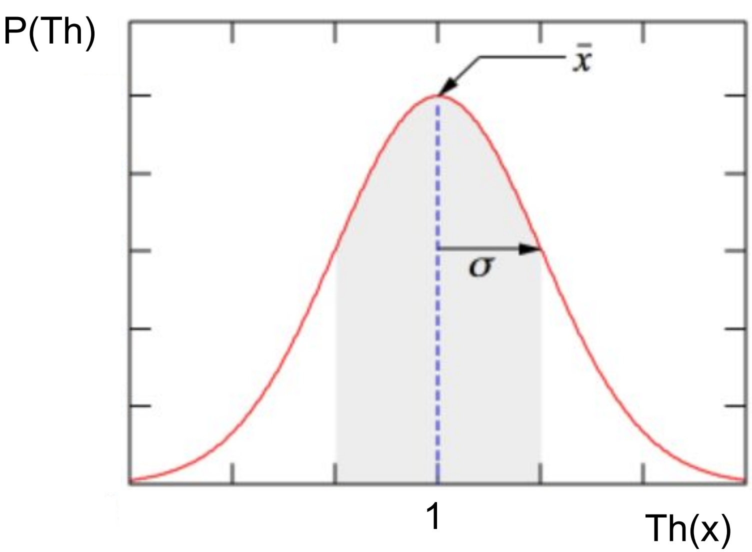
… until the system reaches a new stable state

OFC model

Introduction to the disorder

σ

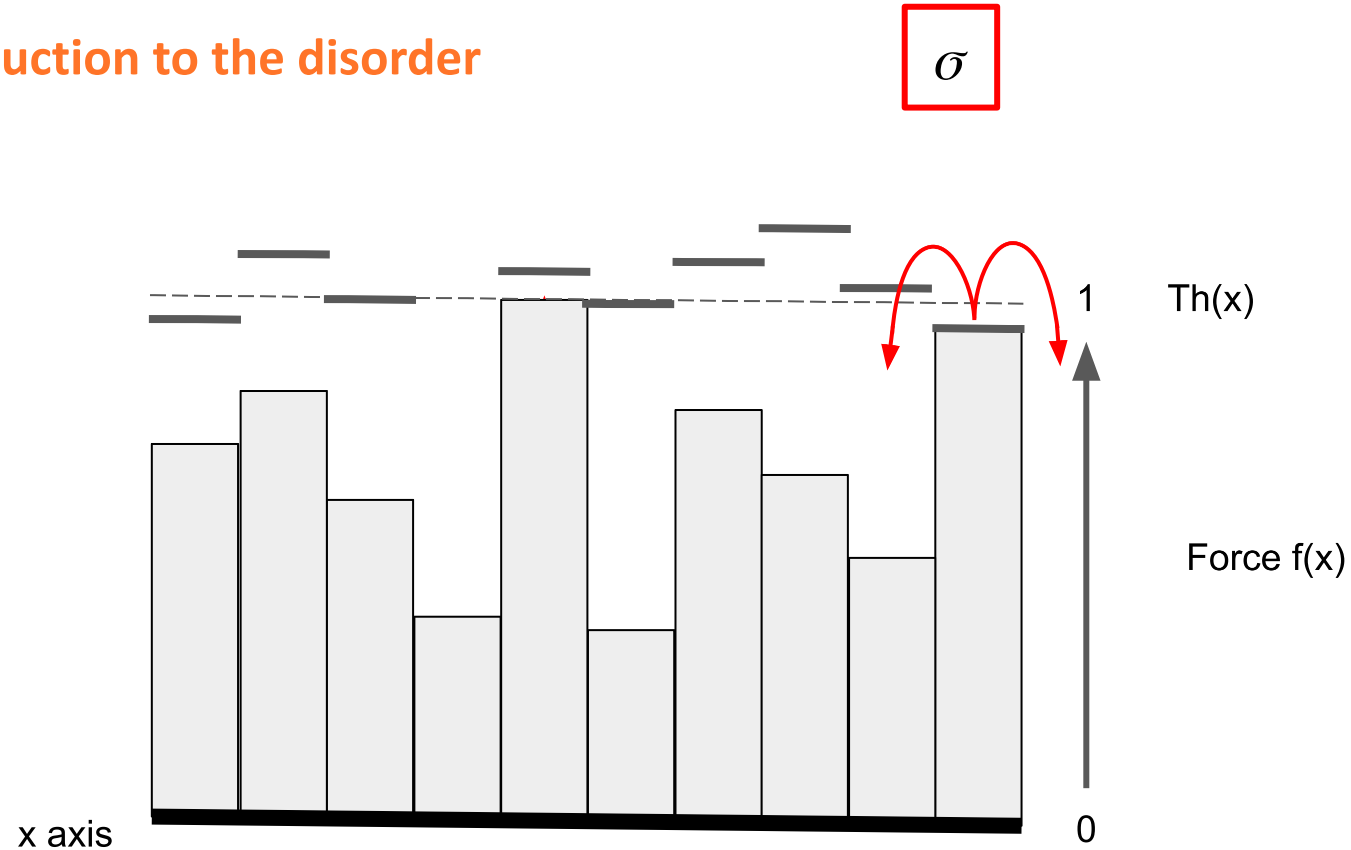




Threshold distribution follow a gaussian around 1

OFC model

Introduction to the disorder

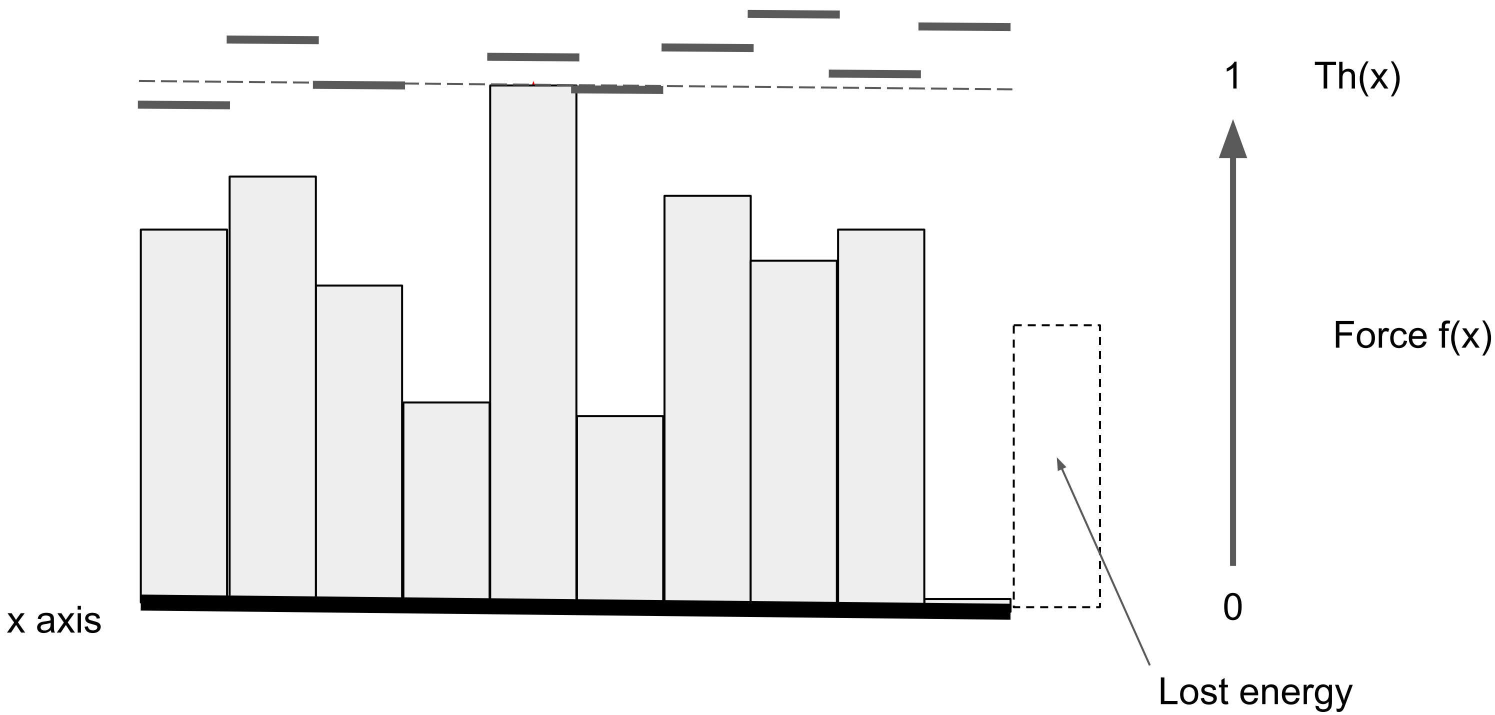


Redistribution of its energy to its neighbors

OFC model

Introduction to the disorder

σ



Open boundary conditions : energy given to the virtual neighbour is lost.

Plan

- **Algorithm optimisation**
- **Dissipation influence on the size distribution**
- **Temporal correlation between avalanches ?**
- **Spatial correlation in the system ?**
- **Robustness with the disorder?**

Plan

- Algorithm optimisation
- Dissipation influence on the size distribution
- Temporal correlation between avalanches ?
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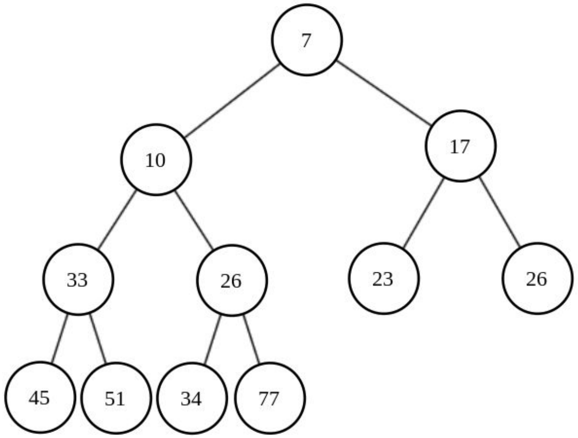
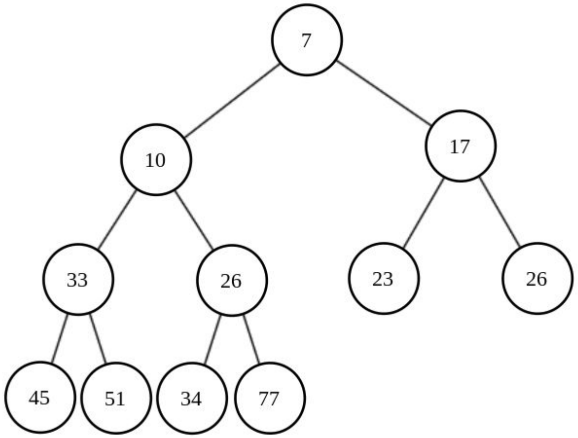
Algorithm optimisation

A new method to find the next center of the avalanche: Heap structure

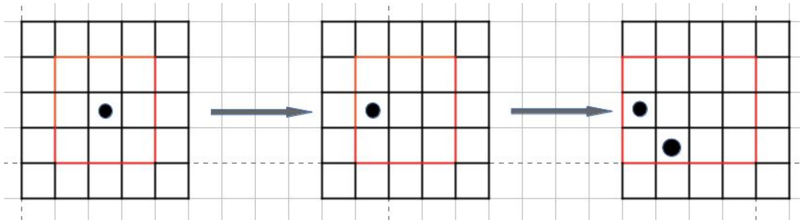


Algorithm optimisation

A new method to find the next center of the avalanche: Heap structure



A box to avoid sweeping the entire system to propagate the avalanche



Algorithm optimisation

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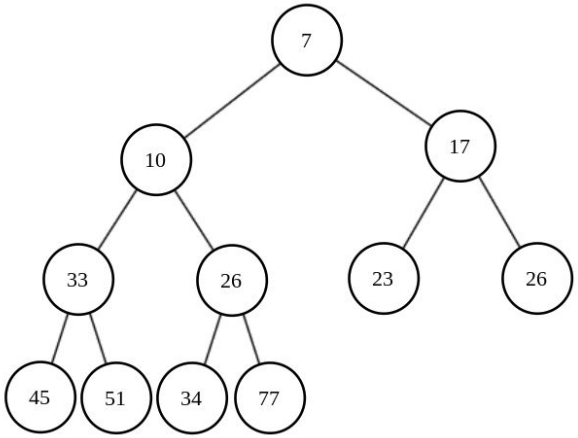
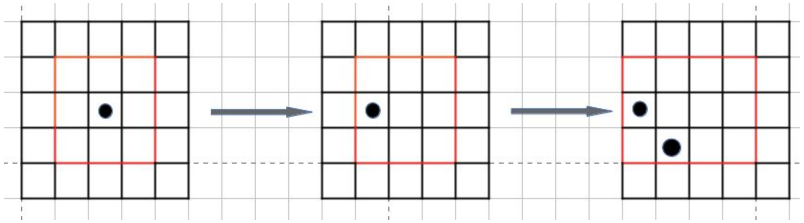
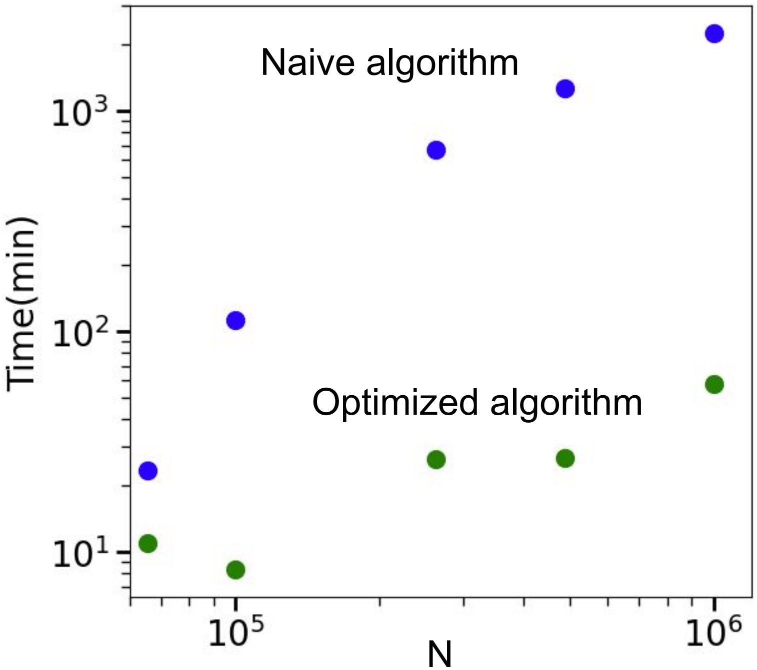


A box to avoid sweeping the entire system to propagate the avalanche



Performances

ν = 12%, 10^7 avalanches



Scaling:

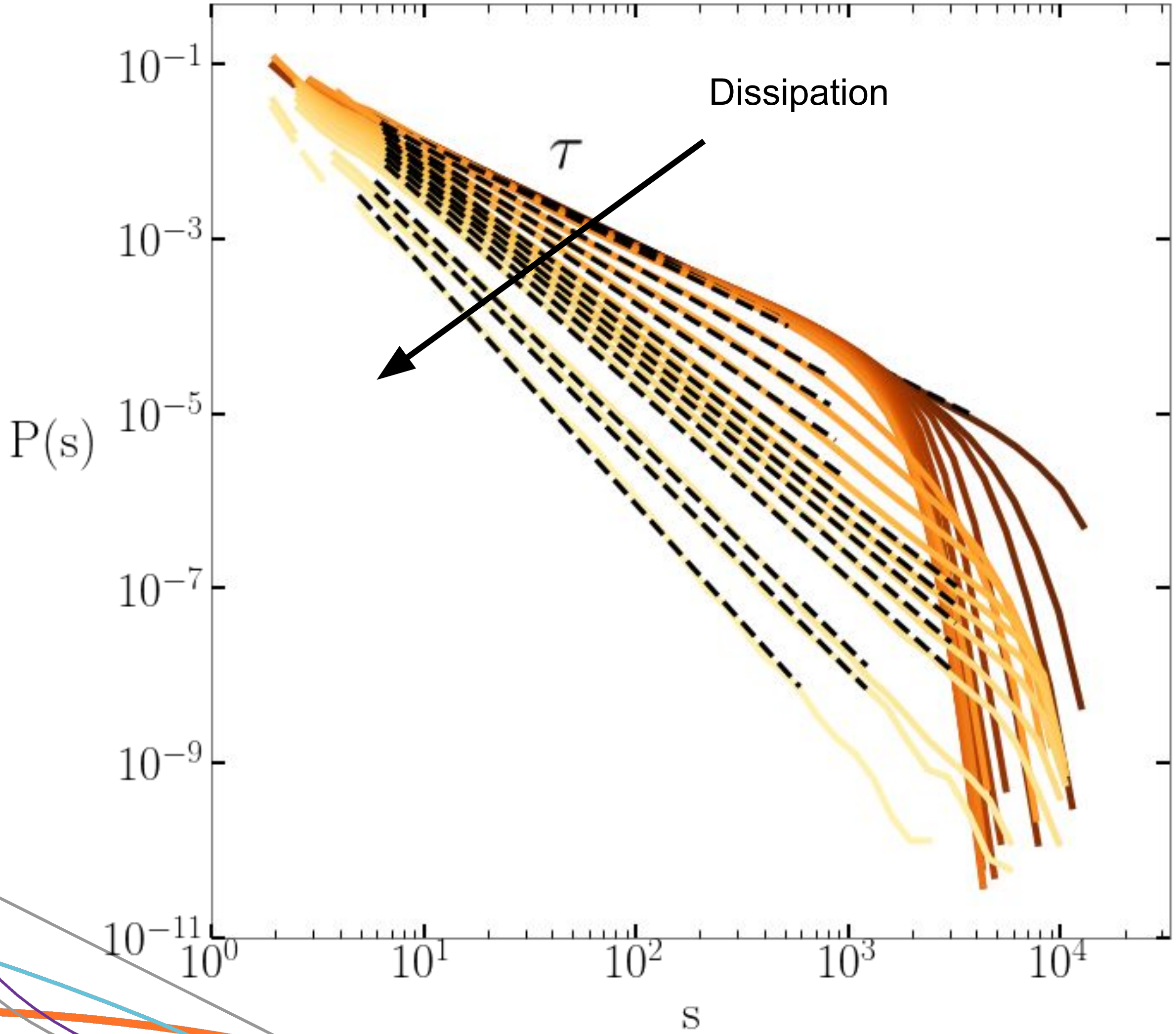
Naive algorithm : O(N)
Optimized algorithm: O(ln(N))

New Plan

- Algorithm optimisation
- Dissipation influence on the size distribution
- Temporal correlation between avalanches ?
- Spatial correlation in the system ?
- Robustness with the disorder?

Simulations

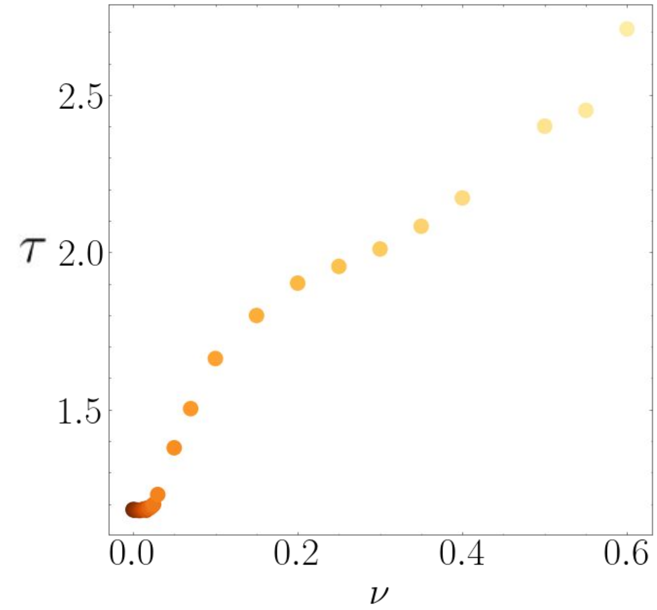
Results: Avalanche size distribution

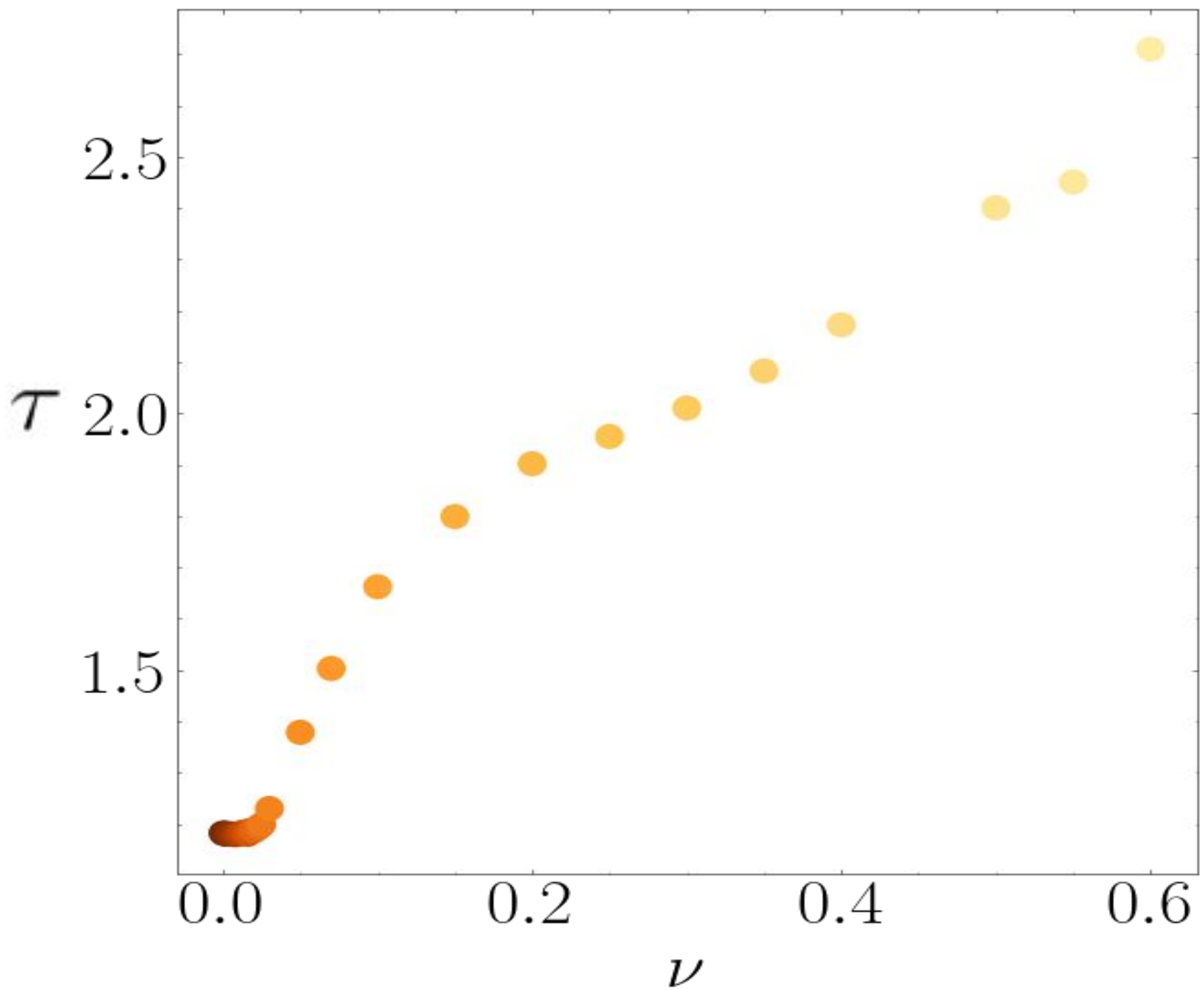


Power law:

$$P(s) = s^{-\tau}$$

Exponent vs dissipation:

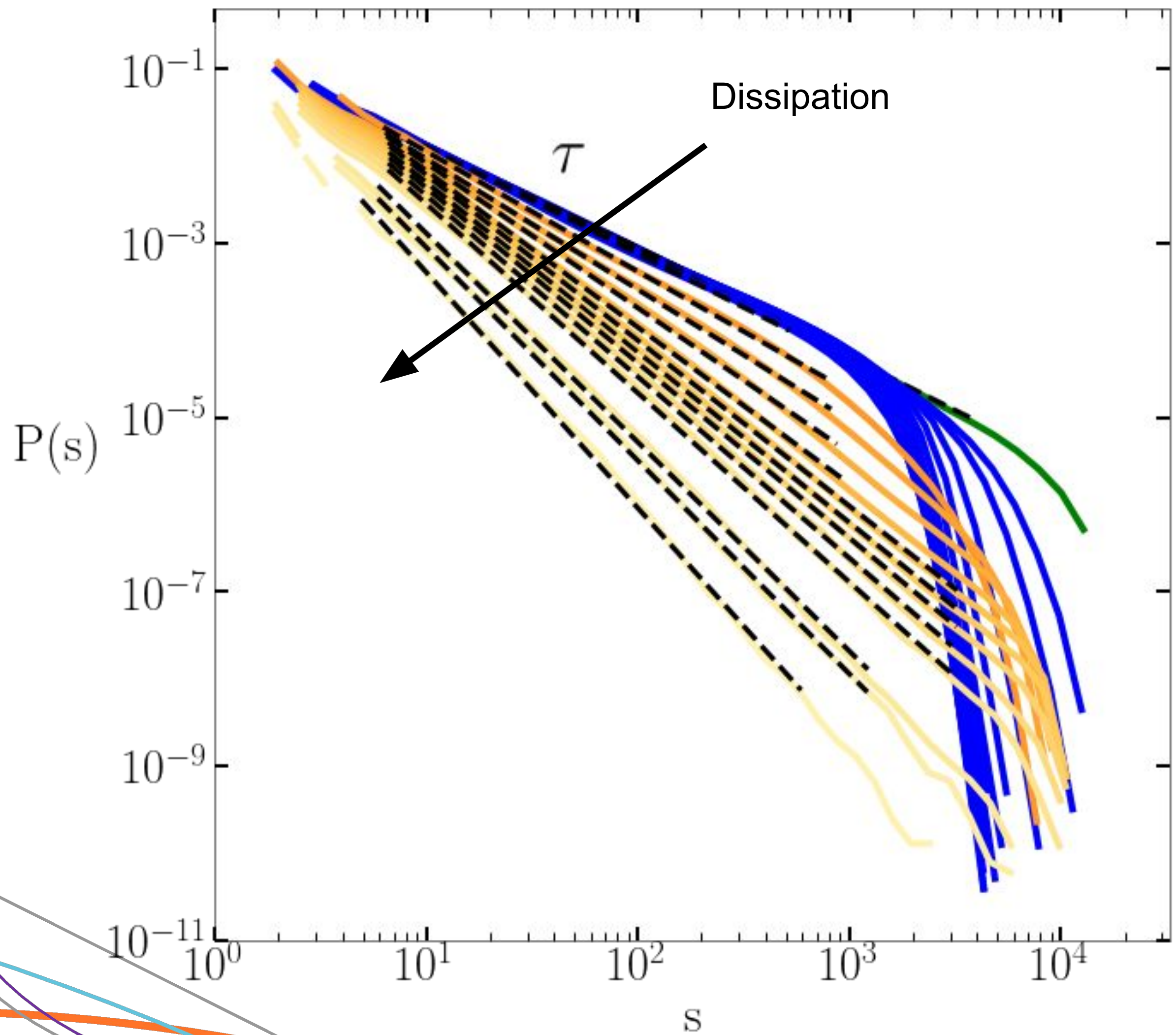




2.5
τ 2.0
1.5
0.0
0.2
0.4
0.6
ν

Simulations

Results: Avalanche size distribution



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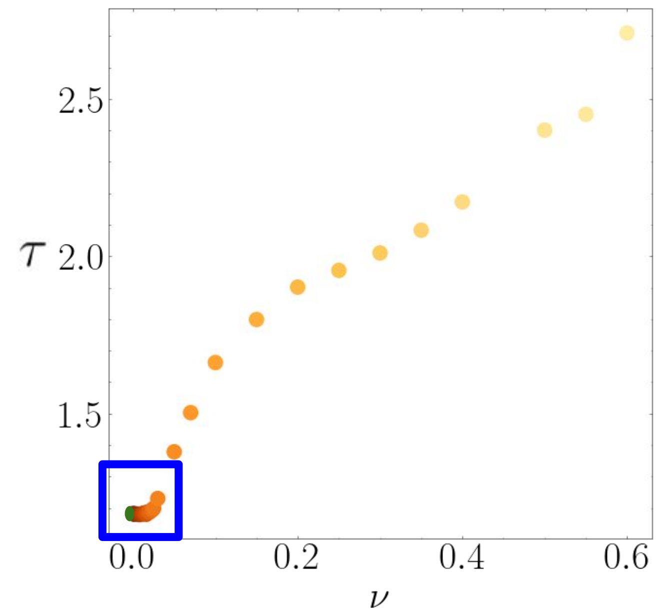
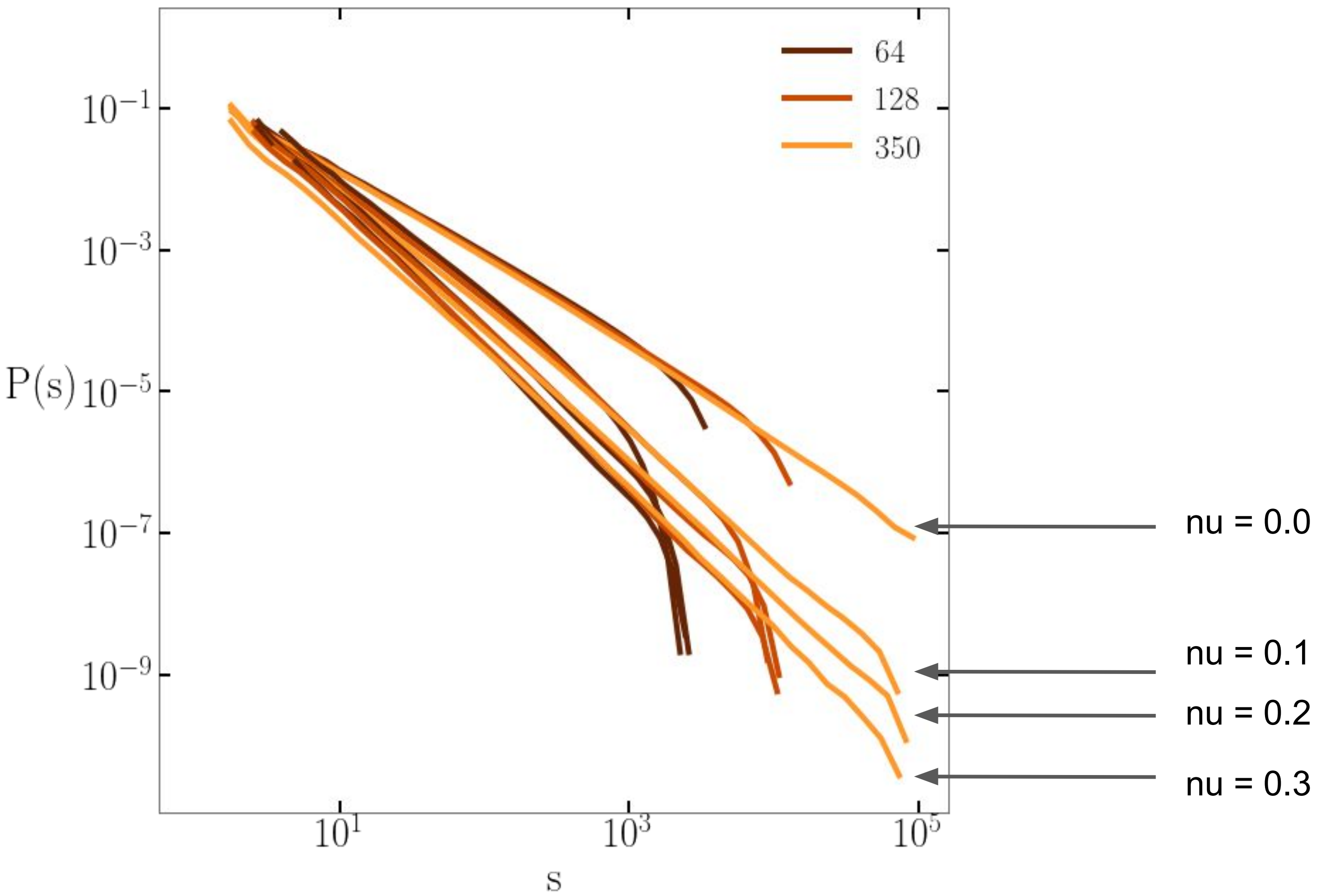


Simulations

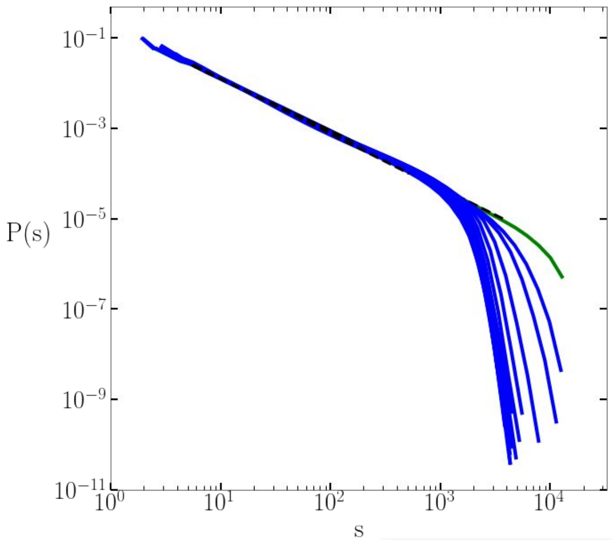
A robuste exponent with the system size



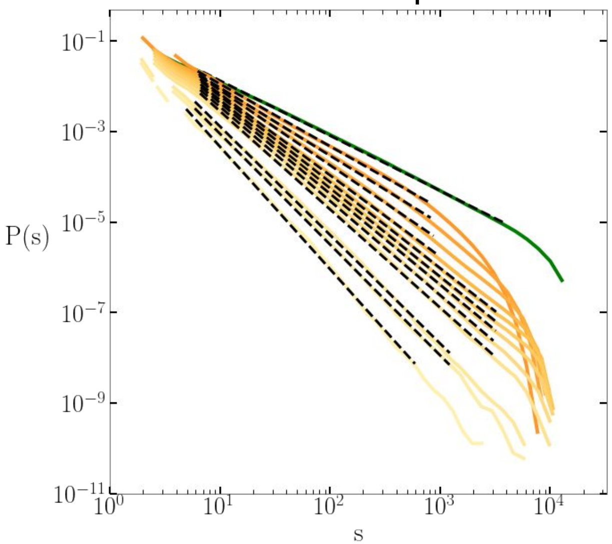
Simulations

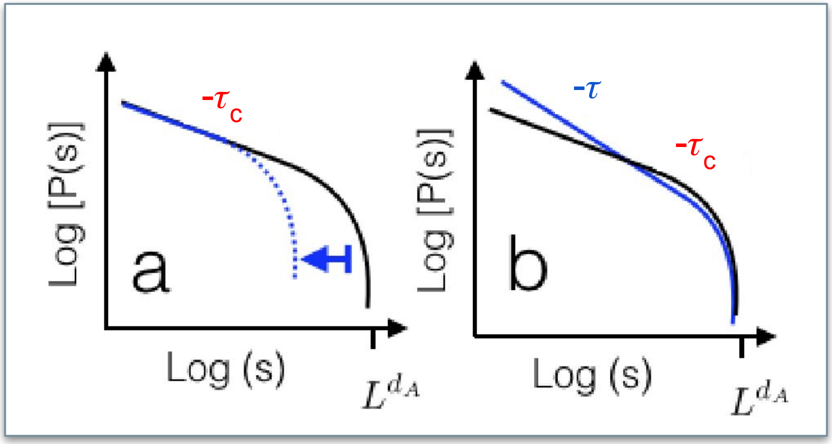
Two regimes : very low dissipation, high dissipation

"Classical view":
Robust exponent and a cut-off



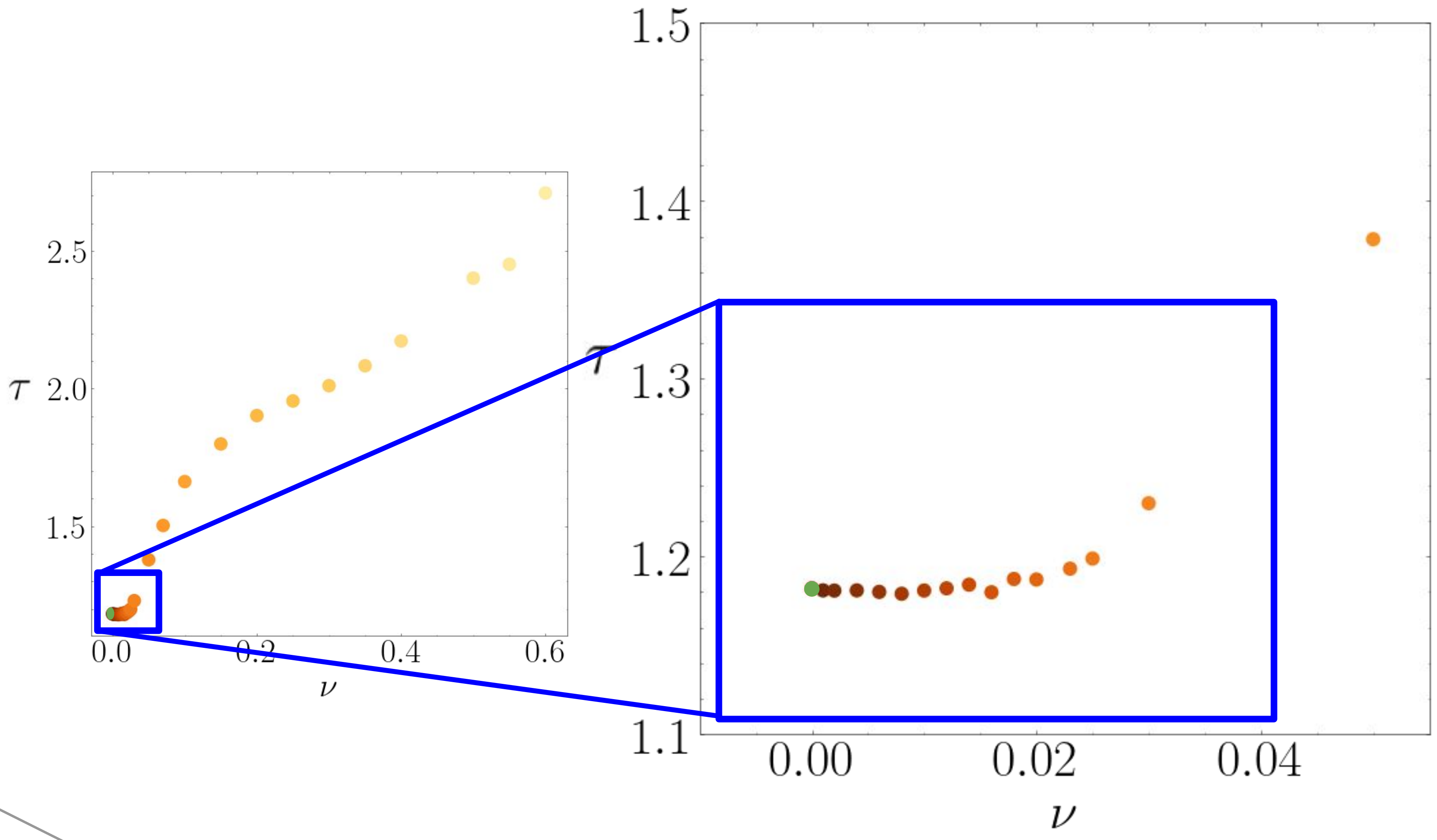
"Anomalous results" :
a variable exponent





Simulations

Zoom at very low dissipation

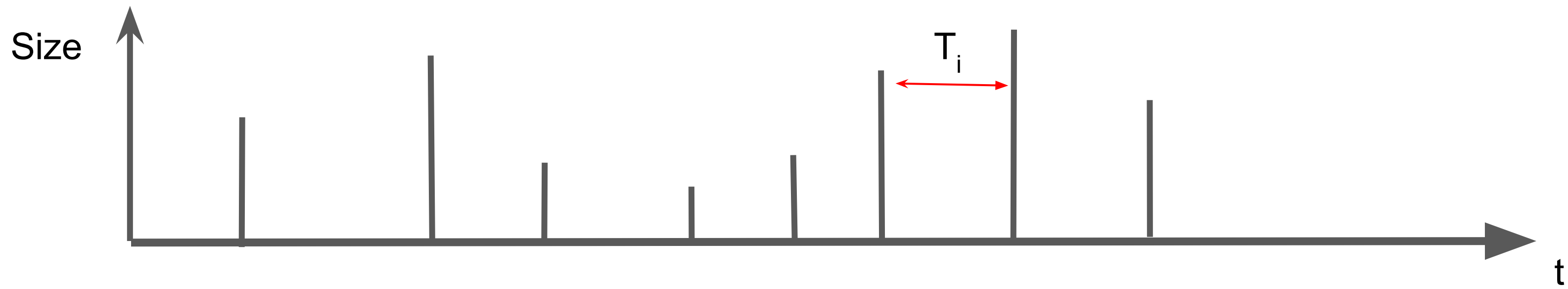


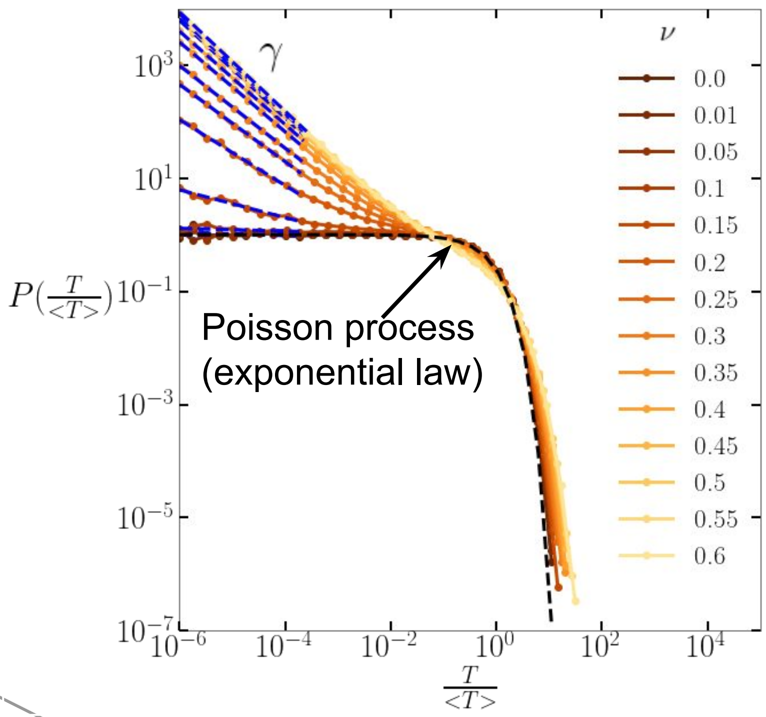
How to try to understand the continuous variation of the exponent as function of the dissipation?

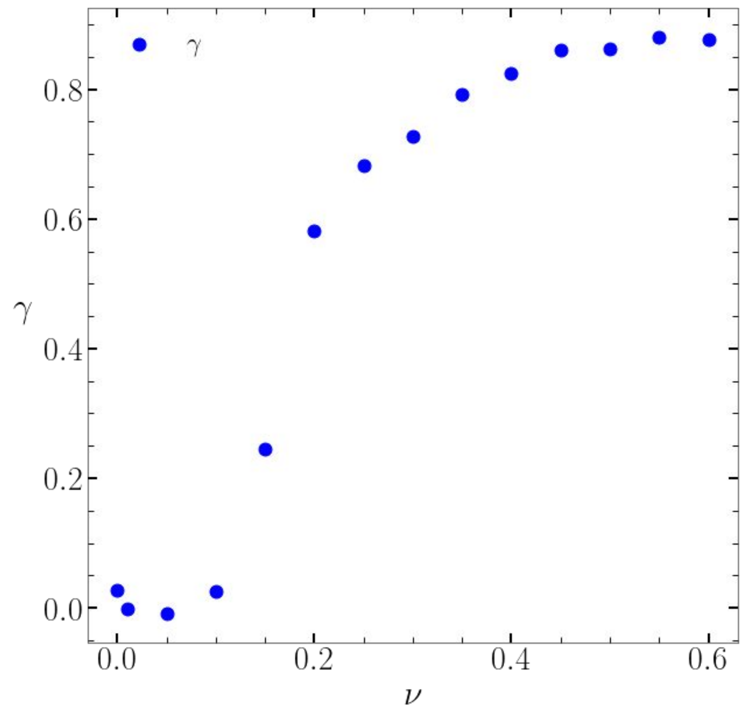
Plan

- Algorithm optimisation
- Dissipation influence on the size distribution
- **Temporal correlation between avalanches ?**
- Spatial correlation in the system ?
- Robustness with the disorder?

Is there a temporal correlation?







Plan

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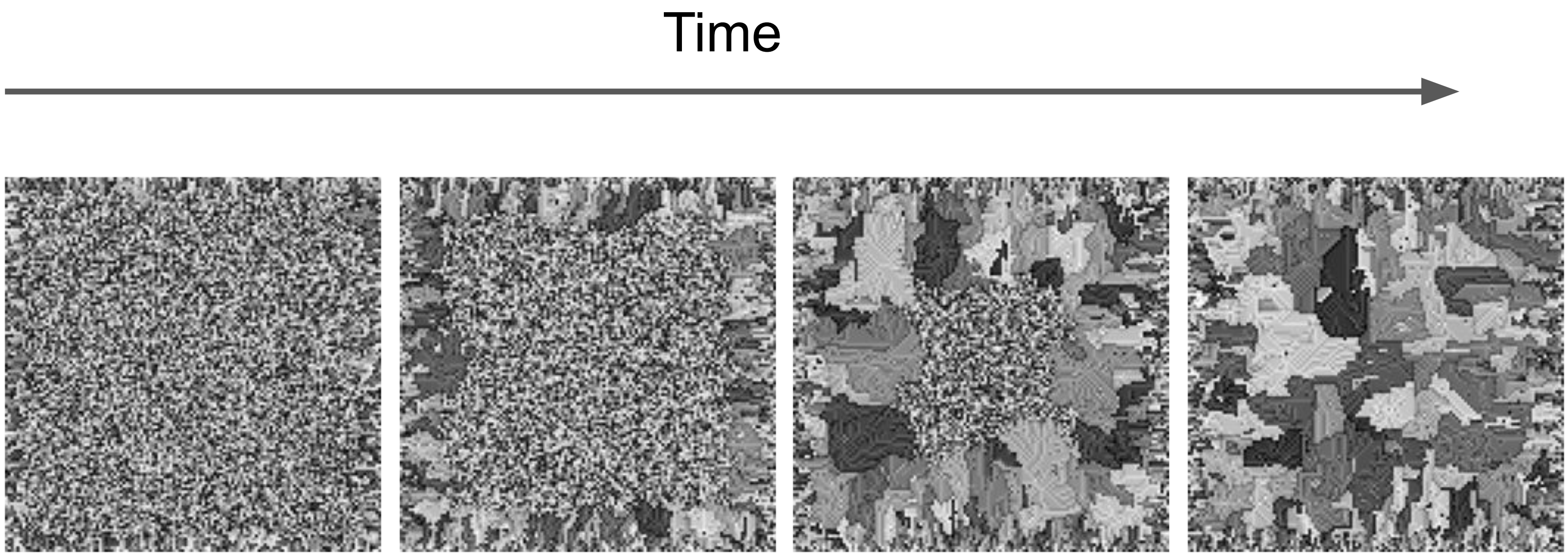
Is there a spatial correlation in the system ?

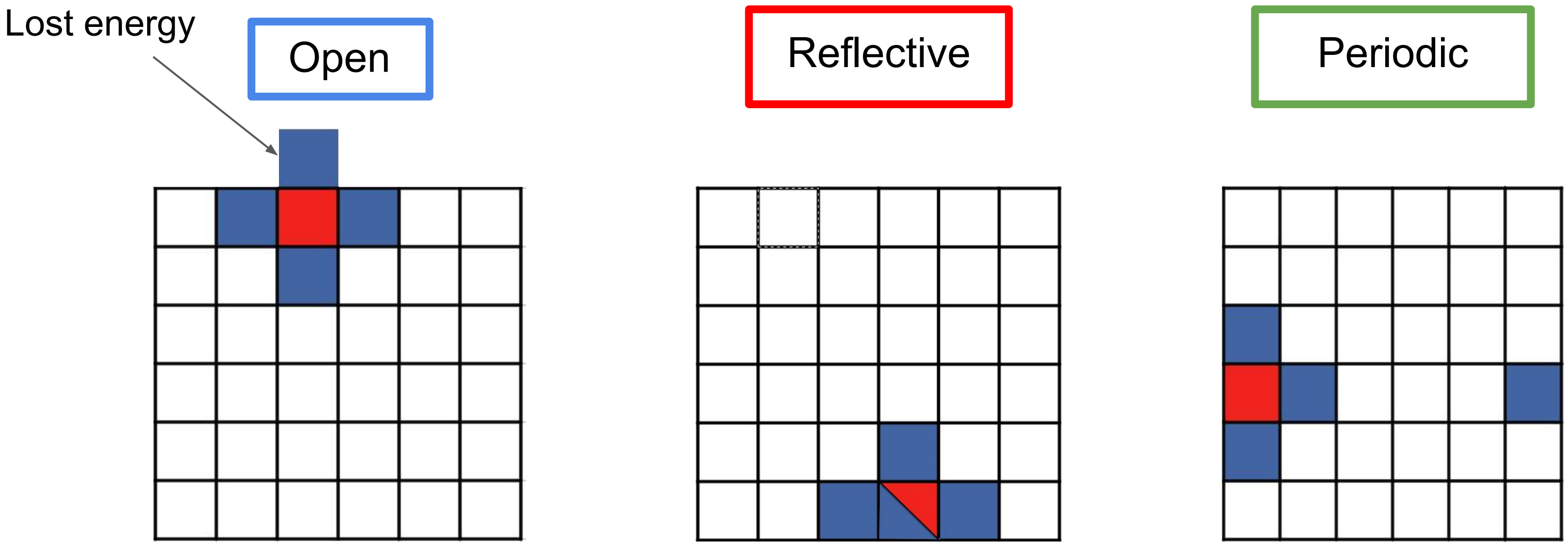
Transient regime



Patches spreading from the border of the system to its center

Is there a spatial correlation in the system ?

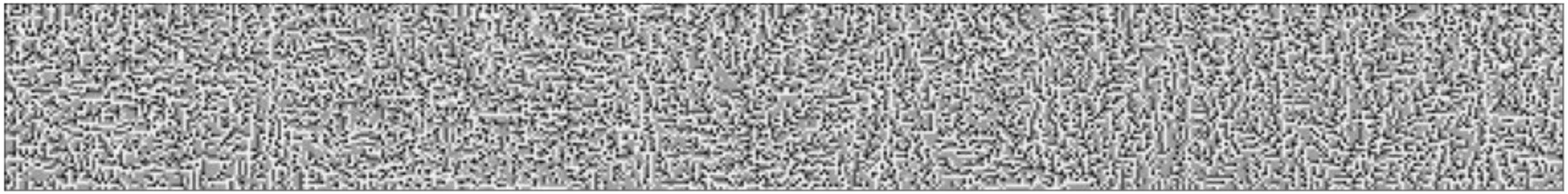
Boundary conditions



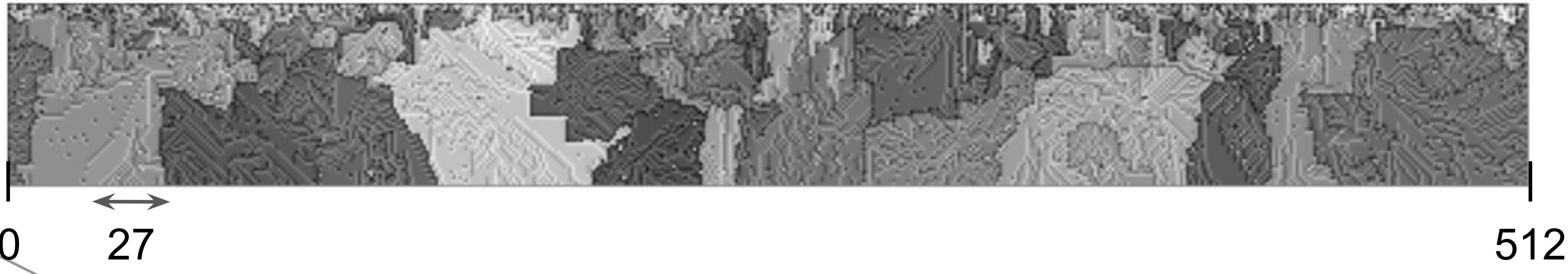
Is there a spatial correlation in the system ?

Hypotheses:
- No spatial correlation in the conservative case
- Structure appearing with the dissipation

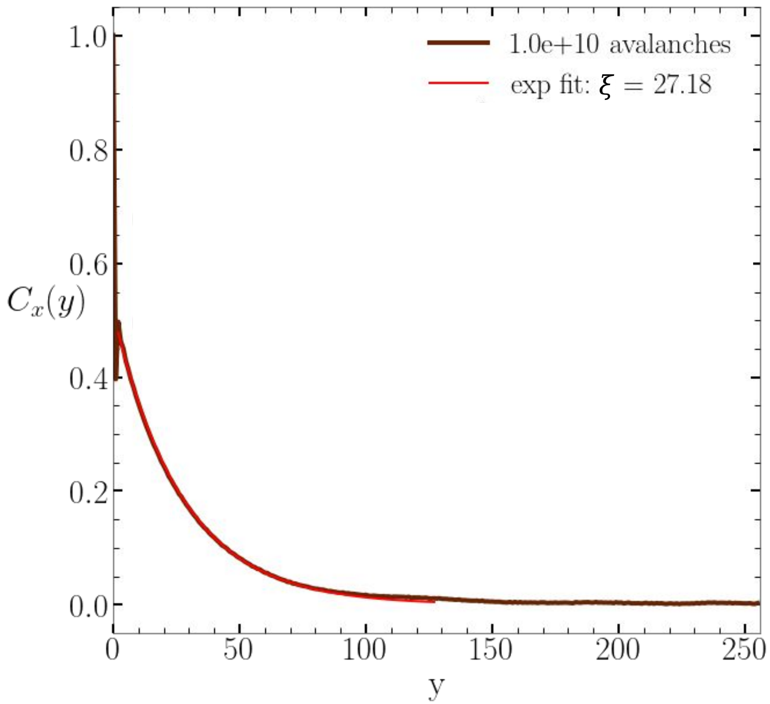
ν = 0%

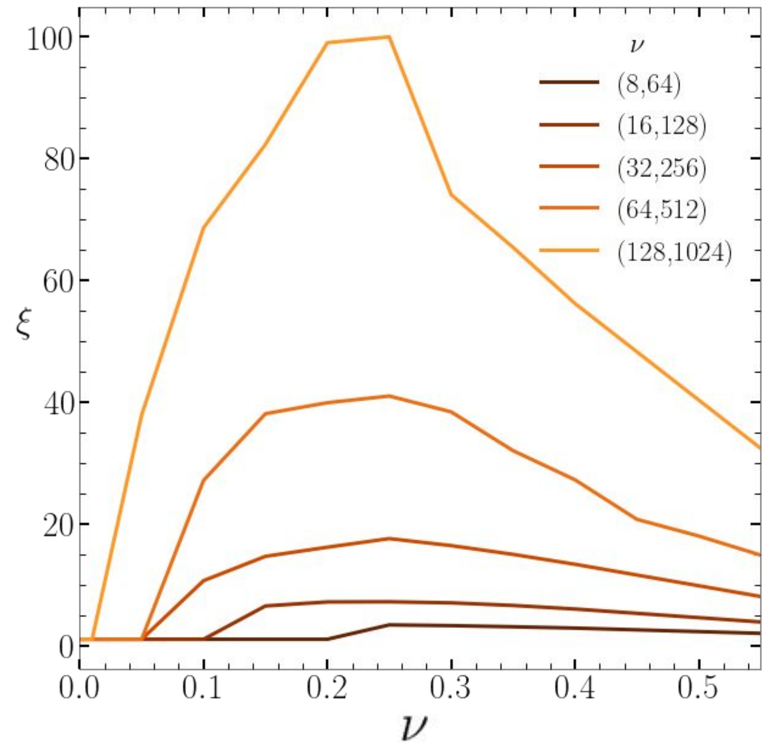


ν = 40%



Is there a spatial correlation in the system ?





Correlation function on the length of the system:

$$C_x(y) = \frac{\langle h_x h_{x,y} \rangle - \langle h_x \rangle^2}{\langle h_x^2 \rangle - \langle h_x \rangle^2}$$

Plan

- Algorithm optimisation
- Dissipation influence on the size distribution
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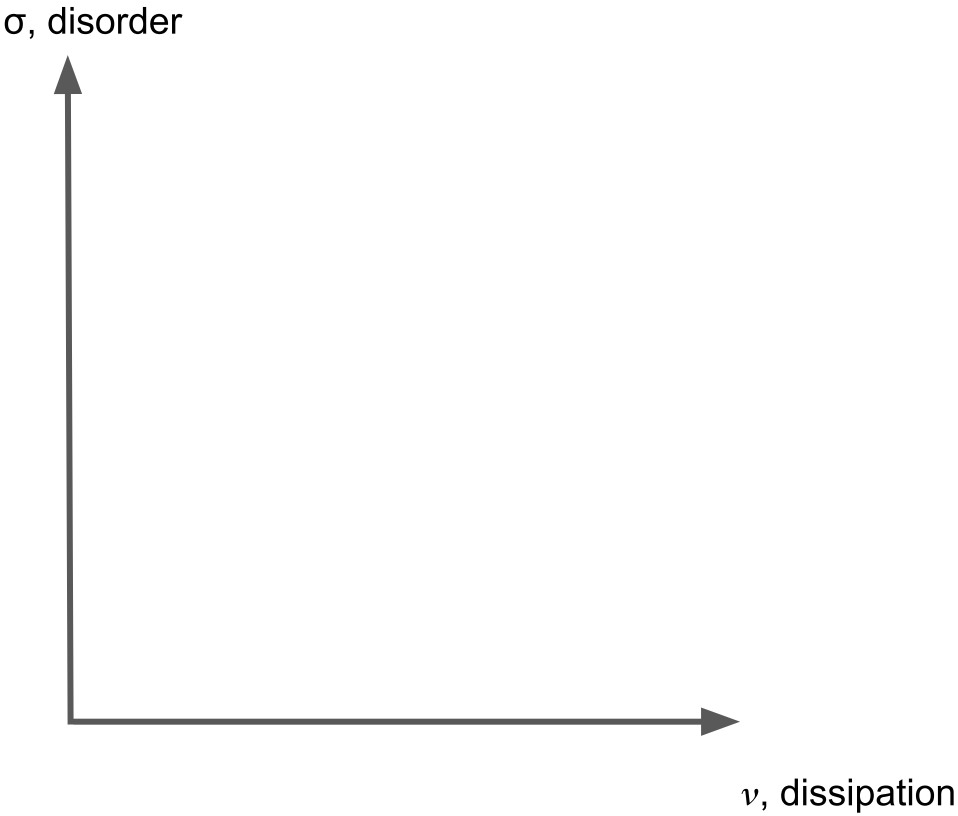
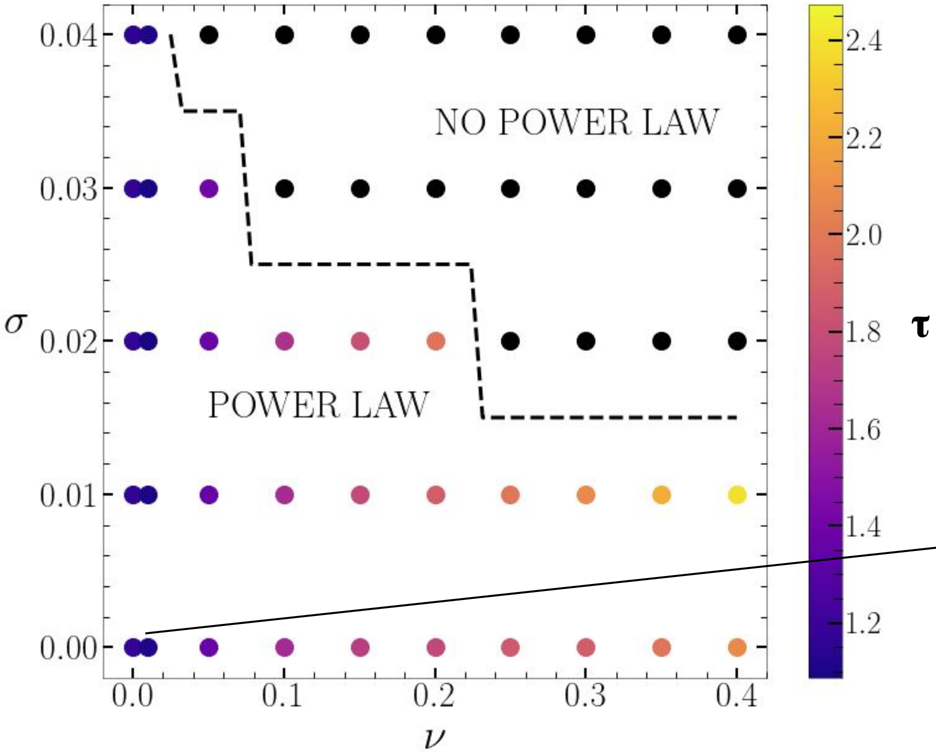
Disorder on the size distribution

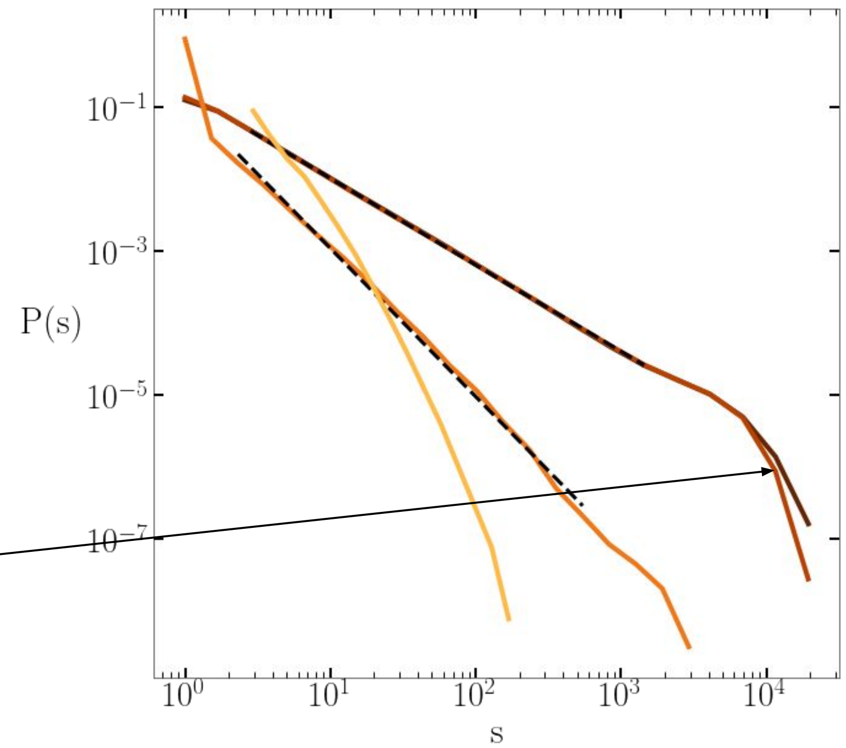
Panorama



Disorder on the size distribution

Panorama

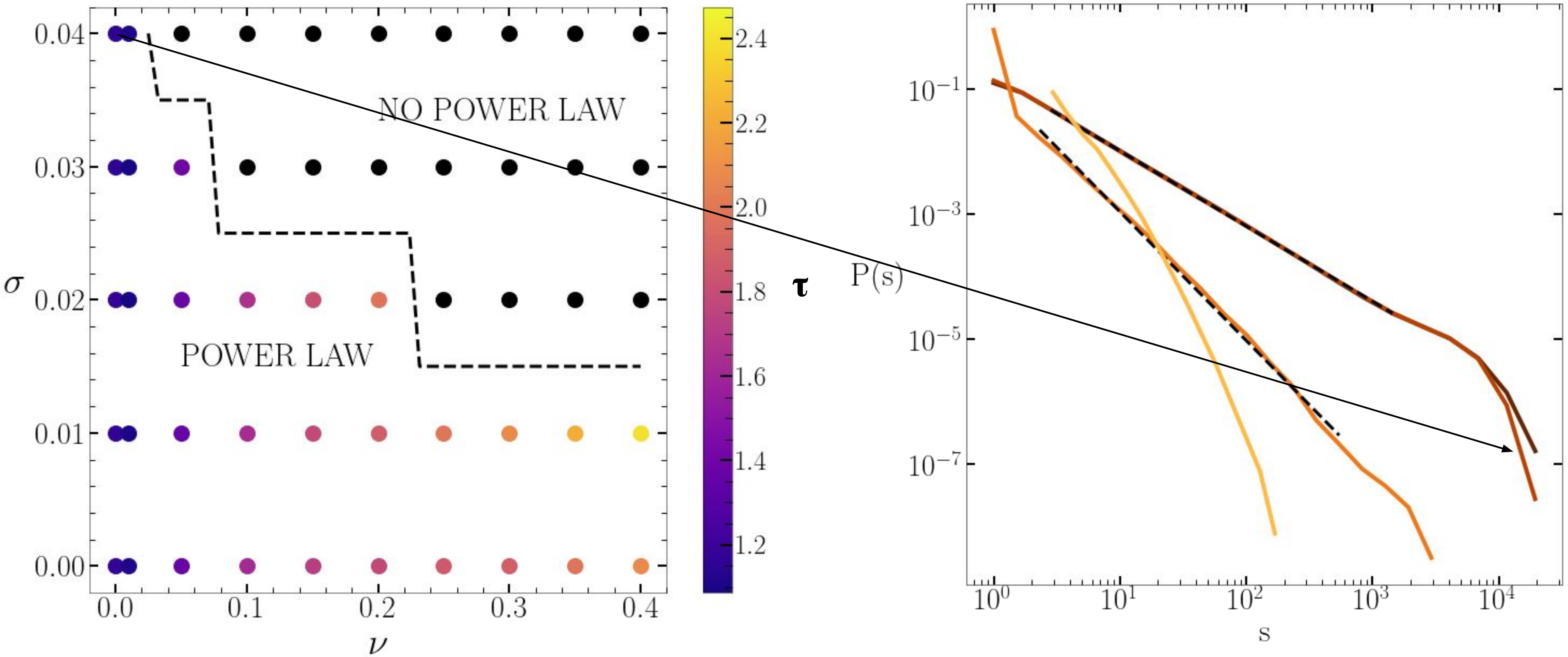




Conservative case : Robuste exponent with the disorder

Disorder on the size distribution

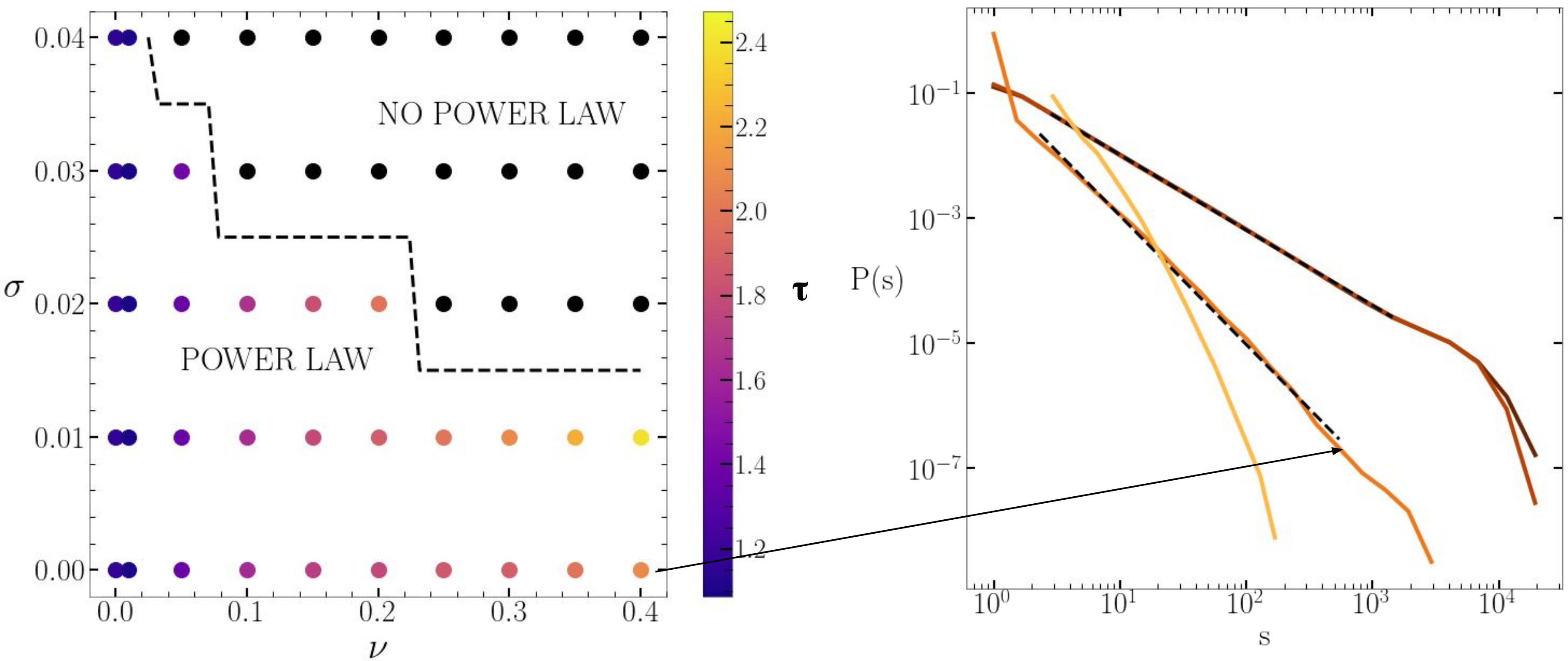
Panorama



Conservative case : Robuste exponent with the disorder

Disorder on the size distribution

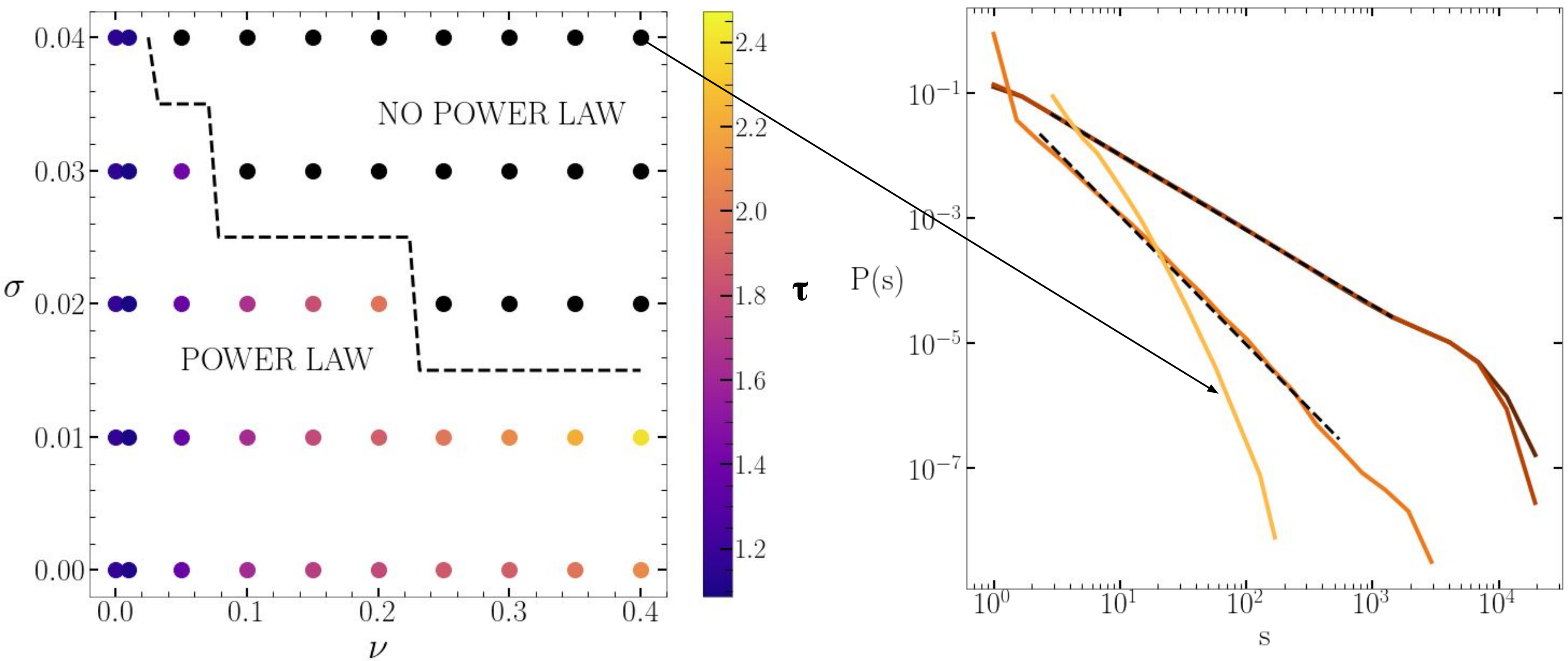
Panorama



The dissipation change the exponent

Disorder on the size distribution

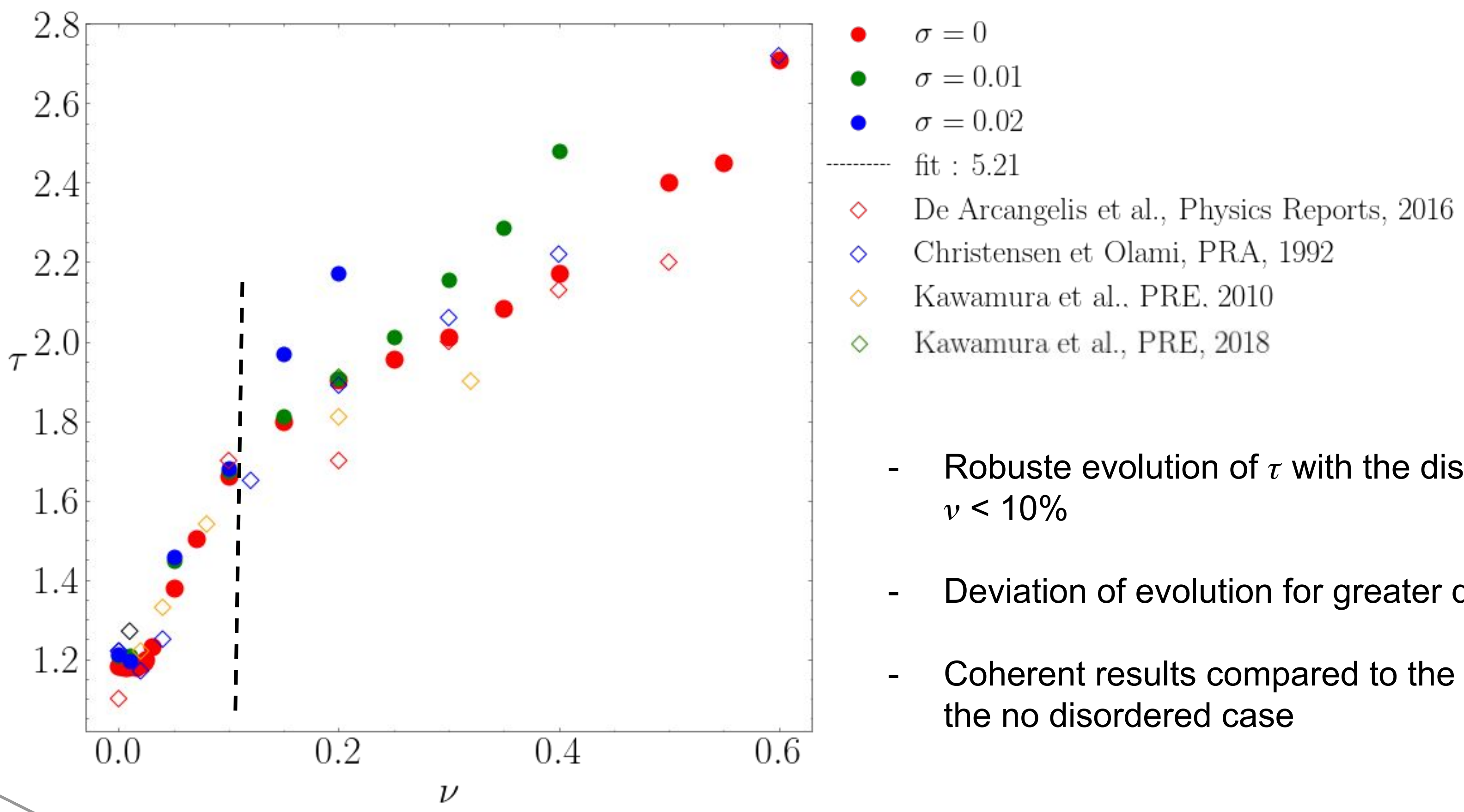
Panorama



Disorder destroys the power law

Disorder on the size distribution

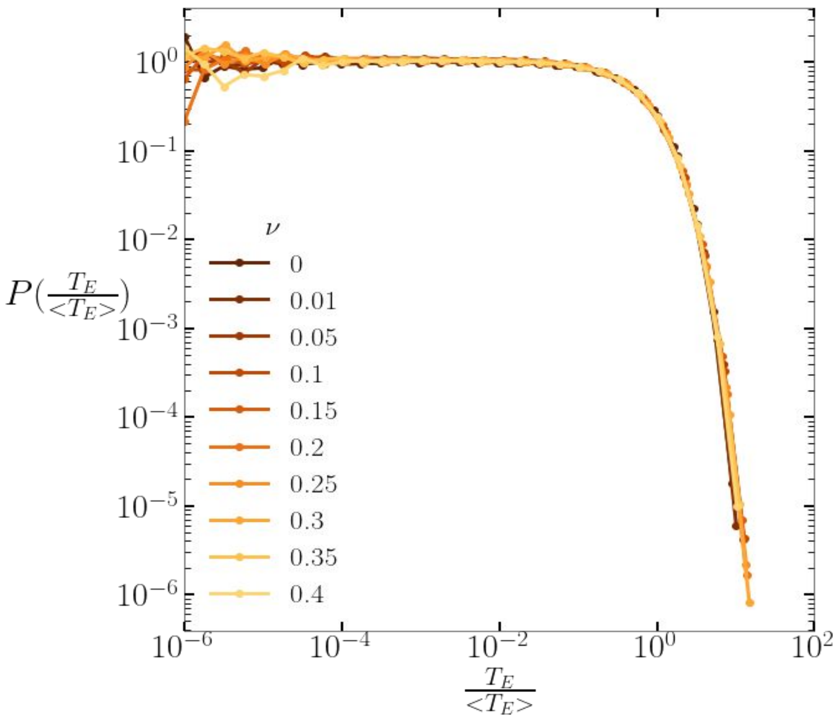
Continuous variation of the exposant τ with the dissipation

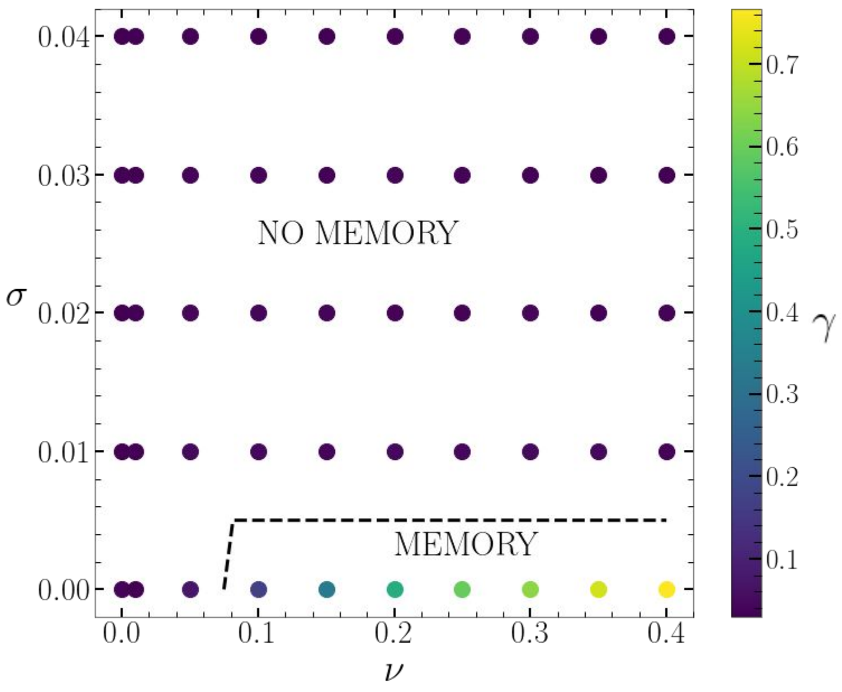


- Robuste evolution of τ with the dissipation for ν < 10%
- Deviation of evolution for greater dissipation
- Coherent results compared to the literature in the no disordered case

Temporal correlation

Memory loss with disorder

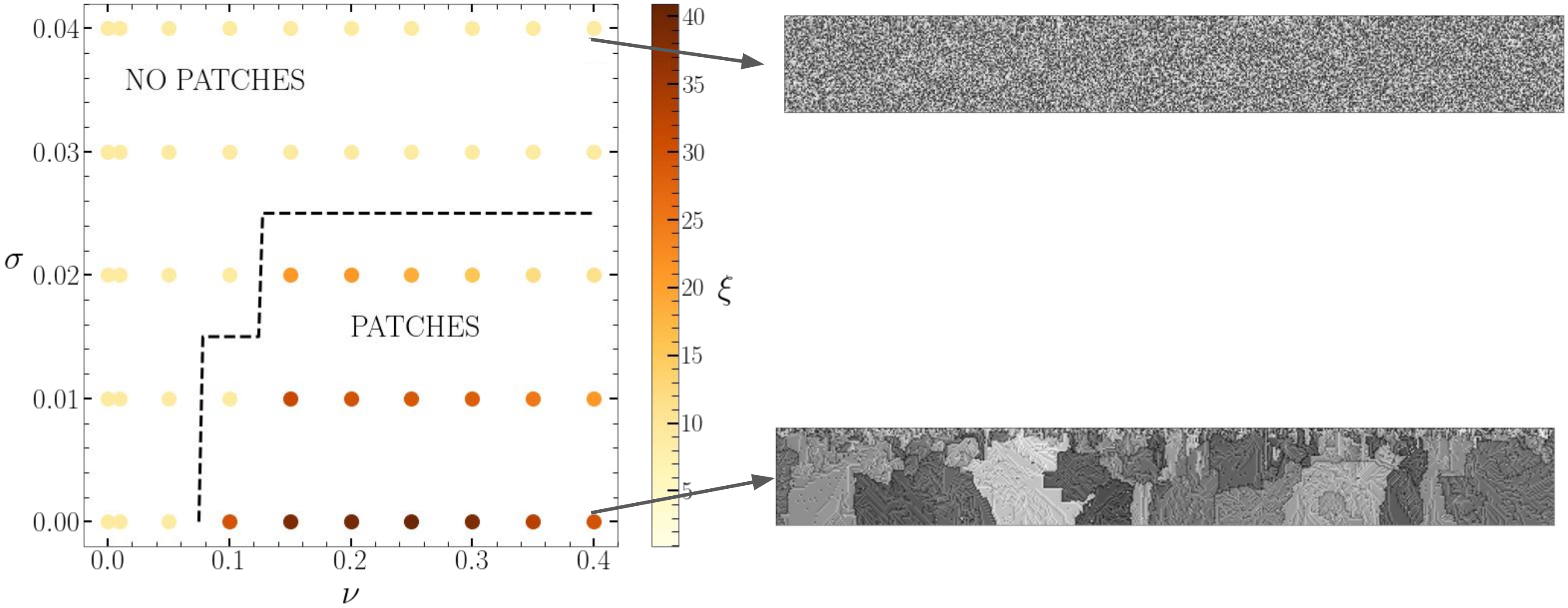




Memory is only present when there is no disorder in the thresholds

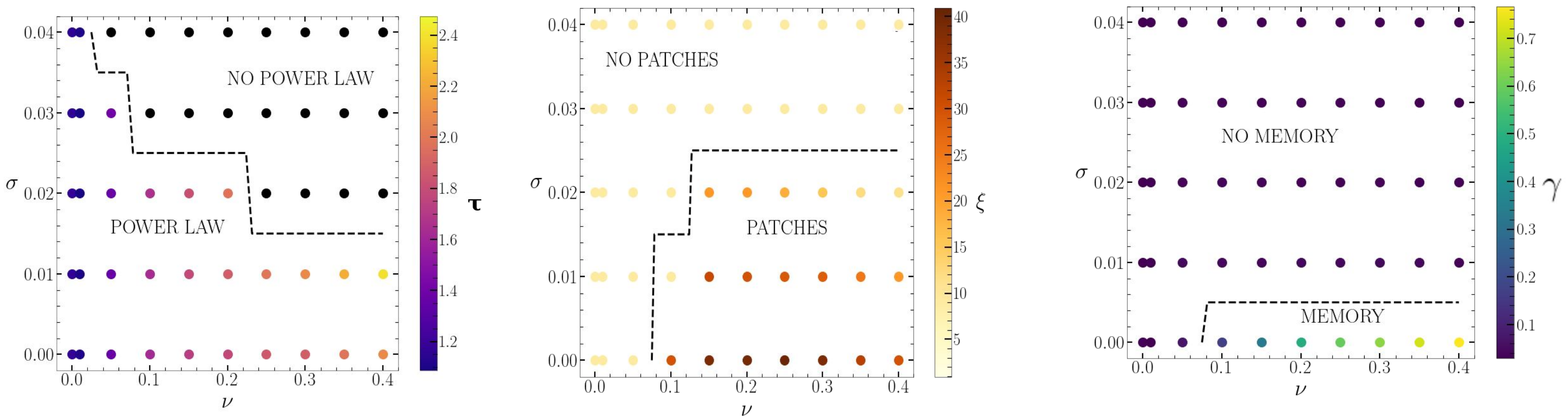
Spatiale correlation

Patch disappearance



Disorder destroy the patches

Conclusion



- Disorder has different impact on τ, γ and ξ.
- Different regions observed depending of τ, γ and ξ.
- Robust evolution of τ at low dissipation with the disorder.
- Two regimes in the size distribution with the dissipation

Thank you for your attention!